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Mathematical Simulation Modeling of the Income Taxation System with the Use of Tukey's Q-Test*

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Abstract. The study deals with the development of a mathematical simulation models for the income taxation system. The paper uses general scientific research methods (analysis, synthesis), mathematical simulation modeling techniques and substantiation of statistical hypotheses. This comprehensive approach is carried out in two stages; this fact distinguished our present study from previously published works on the subject. Flat personal income tax rate is assumed as a basic condition of the system. An effective system of income taxation should take into account two mandatory conditions. The first condition relates to the budget and consists in the non-reduction of tax revenues of the consolidated budget of the Russian Federation. The second – social – condition is to eliminate excessive social inequality in the first five decile groups of citizens by income. In order to fulfill the first condition, we create a mathematical simulation model, which includes non-taxable minimum and tax deductions. In order to comply with the second condition, we propose to use Tukey's q-test, which allows us to assess the degree of social inequality not only in the extreme deciles, but also in their pairwise comparison. We determine that the social condition can be tested with the use of the least significant differences (LSD). In conclusion, we note that our model can be used in the absence of budget constraints. Besides, we propose further directions to develop the methodology and create a system of differential equations that take into account tax, labor and other legislation.

Key words: mathematical simulation modeling, Tukey's q-test, mathematical model, income taxation, social inequality, flat-rate taxation system, model conditions.

* The article is based on the results of studies carried out at the expense of budget funds under the state order given to the Financial University in 2018.

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1. Introduction. Problem statement.

In the Russian Federation, the basis of income taxation is individual income tax levied on the income of residents (earned from the sources in Russia and abroad) and on the income of non-residents (earned from the sources in Russia) according to a proportional (flat) scale[1]. The basic tax rate is 13%. For comparison: in Austria in the presence of progression it reaches 55%, in Belgium and Israel – 50%, in the Netherlands – 52%, in France, Germany and Greece – 45%. An important element of taxation of citizens is tax deductions that represent the amount of citizens' expenses on socially significant or investment purposes and that reduce the tax base (taxable at the rate of 13%, except dividends).

At first glance, the system of citizens' income taxation seems more fair than in the United States and a number of developed European countries. This position is explained, among other things, by the low share of taxes levied on individuals in GDP (less than 4% in Russia, about 10% in the U.S., from 8 to 10% in the EU). However, a more detailed analysis of the main income tax payers reveals that in Russia the important fiscal role of individual income tax is provided by the income inflows from the first four groups of the population by income (in the context of quintiles). On the contrary, in the United States, according to the Congressional Budget Office, about 80% of income tax revenues falls on the 5th quintile group of the population by income, while the 4th group gives another 14% of tax revenues.

Current tax deductions are provided to all taxpayers regardless of their financial situation. A small exception concerns child rearing deductions (1,400 rubles for the first and second child, 3,000 rubles for a third and each subsequent child), the payment of which stops from the month in which the income of the taxpayer receiving it, calculated on a cumulative basis from the beginning of the year, exceeds 350 thousand rubles. Consistently, throughout the year, this deduction can only be received by the taxpayer, whose average monthly salary does not exceed 29.2 thousand rubles. Thus, the right to such an insignificant tax deduction for the family budget for children is not even available to all taxpayers who receive the average salary, which according to Rosstat amounted to 39,1 thousand rubles in 2017. Simultaneously, those who earn considerably higher wages legally get almost all the statutory deductions (social, financial, and investment).

In order to eliminate the situation described above and to smooth social inequality, the Government has repeatedly introduced draft laws on individual income tax reform. However, most of these proposals are aimed at establishing progressive taxation and are based on individual changes in the procedure for calculating individual income tax. Moreover, in accordance with current instructions of the President of the Russian Federation, progressive taxation for individual income tax will not be introduced in the coming years. In our opinion, this provision is justified, first, due to financial reasons – currently Russia's budget is not ready to adopt progressive taxation, the obligatory element of which in all developed countries is a non-taxable minimum. Second, there are quite a few social risks (an increase in the shadow wage market, job cuts), which can bring to naught all the expected positive fiscal effects of this innovation. Finally, third, the establishment of progressive taxation implies mandatory control over citizens' expenses (taking into account their social status), and to exert such control is not yet feasible in Russia.

[1] In the context of the present study, the concepts of "flat" and "proportional" scale of taxation are used as synonyms.

Otherwise, progressive taxation would further exacerbate the issues of social justice with regard to national income taxation.

Thus, the most effective and promising way to improve individual income taxation is to change the flat taxation scheme both in terms of tax rate and certain elements of individual income tax. The introduction of a non-taxable minimum and the improvement of the system of individual income tax deductions can serve as the tools that can help ensure hidden progression and increase justice in taxation on the basis of redistribution of income of different population groups. The proposed changes should be comprehensive and they should affect the entire system of income taxation and at the same time take into account the interests of all those who participate in tax relations. We find it very relevant to use integrative methods of modeling of this system, in particular with the use of mathematical techniques. M.Yu. Andreev notes that "with the help of models it became possible to understand the internal logic of development of economic processes, the logic hidden behind the visible and often seemingly paradoxical picture of economic phenomena, which did not fit into the known theoretical schemes. An experience of using the models has shown that they serve as a reliable tool for analyzing macroeconomic regularities and for forecasting the implications of macroeconomic decisions, provided that the existing relations are maintained. It can be said that a whole "chronicle" of Russian economic reforms expressed in the language of mathematical models was formed [1].

In this regard, the goal of our study is to develop a mathematical simulation model for flat income taxation; the aim of the model is to reduce social inequality and provide stable inflow of income tax revenues in the budget of the Russian Federation.

2. Literature review.

It should be noted that mathematical modeling tools are widely used in the analysis of taxation of income of individuals. We reviewed scientific literature in this field and identified the most controversial aspects in the application of mathematical methods:

– the ratio of the fiscal burden on capital and labor income as a factor in income tax efficiency [2; 3; 4];

– the impact of progressive taxation on total inflows to the state budget [5];

– evaluation of the shadow income market [6; 7].

A. Petrucci used mathematical modeling to estimate the tax rate depending on the tax system's orientation toward capital and labor income [8]. In particular, he built two models: the first is focused on the taxation of financial capital only, the second – on the taxation of capital and labor income. The use of mathematical modeling allowed Petrucci to determine the optimal ratio of state incentives to the taxation of the corresponding type of individual income, depending on the model selected. However, Petrucci's approach does not make it possible to assess how taxes implement the social function, since the approach does not take into account the internal structure of the population by income. A similar study regarding the correlation between the tax burden on capital and labor income was conducted by Ch. Tran [9], who also concluded that the combination of taxation of capital and labor income provides the greatest fiscal effect for the state.

A significant amount of research is devoted to the use of modeling tools to assess the impact of establishing progressive income taxation. For example, Ching-Chong Lai, Chih-Hsing Liao used a mathematical model to assess the impact of a complex progressive scale on the total revenues of the state budget [10]. The scientists proved that, using the Ricardo-Barro model that takes into account the future expectations of the population under deferred taxation, it is possible to determine the Pareto efficient income taxation.

An interesting approach to assessing the fairness of taxation is proposed by E.Yu. Liskina [11]. She uses mathematical modeling to show that the effective tax burden on labor is inversely proportional to the income received. At the same time, her methodology is controversial, because she uses the total burden on labor (individual income tax and insurance contributions) as an effective feature of the model, while these taxes in Russia are levied both on the employee and the employer.

R.O. Smirnov applied a game theoretic model (the problem of decision-making under uncertainty), the basis of which had been laid by S.V. Chistyakov [12], to justify the scale of progressive income taxation [13]. The advantage of the author's model is that it takes into account a sufficient condition of the mathematical function (progressive tax rate) and a necessary condition (maximization of budget revenues). At the same time, this model does not allow us to assess the elimination of social inequality, since in fact it is aimed at finding only the upper and lower limits of the income tax scale.

Of particular interest is an innovative approach to assessing the dependence of the number of cases of tax evasion and the degree of social inequality [14]. The authors use a kinetic model described by a set of nonlinear ordinary differential equations and prove that the effect of tax evasion consists in reducing the size of middle classes and increasing the size of poor and rich classes. Yu.V. Sibiryanskaya and M.B. Kondratenko used economic and mathematical modeling to build a model of the tax system for Ukraine; their model contributes to an increase in the tax burden on wealthy citizens and the withdrawal of shadow income in the legal tax field [15]. However, the use of only the "decision-making" package for analysis is debatable, since the construction of the model in MS Excel services is possible only on the actual data. In their study, the authors made several very significant assumptions (for example, the share of the shadow sector), which may question the results of such an analysis. However, given an appropriate statistical validation of the results, this approach can be effective. In this regard, we can mention an interesting study carried out by Yu.B. Melnikov concerning the adequacy of mathematical and econometric models [16]. In particular, in our present work, we used the reference approach, substantiated by Melnikov, to the assessment of social inequality after the adoption of tax novelties.

It is worth noting that, in contrast to the considered methodologies for modeling the functions of taxation, we use a synergetic approach based not only on the mathematical modeling of the necessary condition (non-reduction of budget revenues), but also on the statistical verification of the sufficient condition (reduction of social inequality).

3. Research methodology.

The paper uses general scientific research methods (analysis, synthesis), and mathematical simulation modeling techniques. Special attention is paid to the application of statistical methods for testing hypotheses using Tukey's Q-test. We present an integrated approach consisting in the step-by-step application of mathematical and statistical tools. This is a feature of the present study in contrast to the works previously published in this field.

The methodology of our research is based on theoretical developments in the field of income taxation. In particular, the constructed simulation mathematical model must meet the following conditions:

1. The budget condition consisting in the absence of decrease in tax receipts in the budget of the Russian Federation.
2. The social condition that consists in reduction of social inequality of citizens and their stratification by the level of income. In this aspect, we should note that it is impossible to reduce the existing gap between the extreme income deciles by 15.3 times with the help of taxation tools alone. At the same time, it is quite possible to redistribute citizens' incomes in the first half of deciles by means of fiscal tools.

Previously, we noted that the budget condition is a necessary one. Despite the fact that the debated in the field of income taxation usually boils down to justice, financial stability in the current economic conditions is a priority of national policy *(Tab. 1)*.

The formation of two conditions determines the presence of two stages in the simulation modeling system. A mathematical model is used to fulfill the budget condition, and statistical testing of hypotheses using Tukey's q-test is used to fulfill the social condition.

The use of a mathematical simulation model is justified by the fact that as a result of multiple changes in the parametric characteristics it is possible to predict various options for the development of the income tax system. The advantage of this approach consists in the fact that it makes it possible to apply time simulation to the objects on which real experiments are difficult to perform or impossible to implement in principle [17]. Moreover, new parameters can be added to the simulation model subject to their appropriate economic justification.

The choice of the statistical criterion is justified by economic prerequisites in the content of the category "social inequality". In many literature sources, it is estimated primarily by indicators such as the Gini index [18; 19] and R/P 10% ratio (determined by the differences between the extreme decile groups of citizens by income) [20]. In our opinion, a truly effective governmental policy should be aimed at equalizing incomes for all ten deciles, because the assessment of extreme groups of citizens seems to us not objective enough. Among the set of statistical criteria for substantiating the hypotheses, only Tukey's Q-test helps estimate the equality of values in more than two samples.

Table 1. Key indicators of the consolidated budget of the Russian Federation, billion rubles

Indicator	2014	2015	2016	2017	2017 to 2014, %
Public debt of the Russian Federation (at the end of the year)	10 299	10 952	11 110	11 560	112
State external debt of the Russian Federation (at the end of the year)	3 058	3 644	3 106	2 870	94
State internal debt of the Russian Federation (at the end of the year)	7 241	7 308	8 003	8 690	120
Reserve Fund	3 121	4 426	3 421	913	29
National Welfare Fund	4 388	5 227	4 359	3 753	86
Deficit/surplus of state extra-budgetary funds	-26	-680	-185	44	-172
Deficit/surplus of consolidated budgets of constituent entities of the Russian Federation	-448	-172	-13	-52	12
Federal budget deficit/surplus	-334	-1 955	-2 956	-1 331	399
Source: Own calculations based on data of the Ministry of Finance of Russia. Available at: https://www.minfin.ru/ru/ (accessed: 20.07.2018).					

In particular, with the help of a mathematical model it is possible to determine how our following proposals meet the budget condition:

1. Establishment of a non-taxable minimum income in the Russian Federation at the level of the subsistence minimum for the able-bodied population (regional level), adjusted for the individual income tax rate. It is assumed that the right to use this minimum will be granted only to those citizens whose average income in the previous tax period did not exceed the subsistence minimum twofold [21].

2. Increase in the amount of standard child rearing tax deductions to the amount of the subsistence minimum per child (regional level) with the restriction of the right to use it by taxpayers whose income for the previous tax period did not exceed the annual average.

3. Restriction of the right to apply social deductions for education for taxpayers whose income for the previous tax period exceeded the annual average threefold.

To calculate individual income tax receipts, we used the following data on the basis of our proposals:

– indicators of the forecast of socio-economic development of the Russian Federation for the next financial year and planning period (wage fund), elaborated by the Ministry of Economic Development of the Russian Federation;

– dynamics of the tax base according to Form No. 5-NDFL;

– dynamics of the tax base according to Form No. 7-NDFL;

– dynamics of actual tax revenues according to Form No. 1-NM;

– tax rates, benefits and preferences under Chapter 23 “Individual income tax” of the RF Tax Code and other sources;

– *Regions of Russia. Socio-Economic Indicators. 2017: Statistics Collection*. Rosstat. Moscow, 2017. 1402 p. (age structure of the population; participation in labor force; average annual number of employed; average monthly nominal accrued wages of employees of the organizations; average per capita monetary incomes; structure of people’s monetary incomes; population with monetary incomes below the subsistence minimum; consumer expenses on average per capita; number of students enrolled in bachelor’s, specialist’s, and master’s programs; number of students enrolled in training programs for skilled workers and employees).

4. Research results.

4.1. Mathematical equation of the budget condition.

In order to build a proportional income tax scale it is necessary to divide all taxpayers into groups in accordance with the flat tax model we developed earlier (by the size of their income – taking into account the right of one group to use a non-taxable minimum; and by the limitation of the right to receive social and standard tax deductions).

Thus, the set of taxpayers should be divided into the following groups:

1) individuals entitled to a non-taxable minimum;

2) individuals entitled to standard child rearing deductions;

3) individuals entitled to social tax deductions for education;

4) other individuals who receive income at the rate of 13% and who are entitled to other tax deductions (except dividends);

5) individuals receiving income at other tax rates, including dividends at the rate of 13% (the number of groups after the fifth one depends on the number of individual income tax rates).

Let us assume that the number of such groups is m (the number of levels). We distribute these groups in order of increasing the average income of taxpayers in the group (presented above) and give them the corresponding index i = 1; 2; ... m [22].

In accordance with current norms of the Russian tax legislation, all Russian taxpayers can be divided into nine levels: the first four groups by income and by rates – 9%, 13% (dividends), 15%, 30%, 35% (m = 9).

Each group has its own taxable base S_{0i} (1):

$$\sum_{i=1}^{m} S_{0i} = S_0 , \qquad (1)$$

in which S_0 is the taxable base determined by current rules.

Under a flat tax scale (flat tax rate n_0), the total individual income tax will be determined as follows (2):

$$C_0 = S_0 \times n_0 = \sum_{i=1}^{m} S_{0i} \times n_0 . \qquad (2)$$

Under a proportional tax scale, taking into account our proposals, we introduce the following symbols:

n_i – new tax rates;

q_k^i – number of taxpayers in each group of the constituent entity of the Russian Federation;

k – serial number of the constituent entity of the Russian Federation;

M_k – size of the non-taxable child rearing minimum in the k-th constituent entity of the Russian Federation;

D_k – size of the minimum child rearing subsistence level in the k-th constituent entity of the Russian Federation;

d_k^i – number of child rearing tax deductions received by taxpayers of groups 1 and 2 in the last tax period in the k-th constituent entity of the Russian Federation;

O^k_i – amount of standard tax deductions received by taxpayers of the i-th group in the last tax period in the k-th constituent entity of the Russian Federation;

C^k_i – amount of social tax deductions received by taxpayers of the i-th group in the last tax period in the k-th constituent entity of the Russian Federation.

Changes in property tax deductions are not included in the model due to the high degree of their proposed differentiation.

Thus, when our own changes are applied, we get the following:

1. The amount of tax payments that has not been received by the budget due to the provision of non-taxable minimum for individual income tax in group 1 is defined as $\sum_{k=1}^{k=85} q_k^i \times 12M_k$.

2. The amount of tax payments that has not been received by the budget due to the provision of standard child rearing deductions for individual income tax in groups 1 and 2 is defined as $\sum_{k=1}^{k=85} d_k^i \times 12D_k$.

3. The additional amount of tax payments formed due to the restriction on the right to use standard child rearing deductions is defined as $\sum_{k=1}^{k=85} (O_3^k + O_4^k)$.

4. The additional amount of tax payments formed due to the restriction on the right to use social tax deductions for education is defined as $\sum_{k=1}^{k=85} C_4^k$.

Under the flat taxation scheme (when the proposed changes and n_i rates are adopted), the total inflow of payments from individual income tax will be determined as follows (3):

$$C_1 = \left(S_{01} - \sum_{k=1}^{k=85} q_k^1 \times 12M_k - \sum_{k=1}^{k=85} d_k^1 \times 12D_k\right) \times n_i + \left(S_{02} - \sum_{k=1}^{k=85} d_k^2 \times 12D_k\right) \times n_i + \left(S_{03} + S_{04} + \sum_{i=3}^{i=4} \sum_{k=1}^{k=85} O_i^k + \sum_{k=1}^{k=85} C_4^k\right) \times n_{i+} \sum_{i=5}^{m} S_{1i} \times n_i . \qquad (3)$$

Thus, to fulfill the budget condition, the inequation for the simulation mathematical model of flat taxation should have the following form (4):

$$\sum_{i=1}^{m} S_{0i} \times n_0 \le \left(S_{01} - \sum_{k=1}^{k=85} q_k^1 \times 12M_k - \sum_{k=1}^{k=85} d_k^1 \times 12D_k\right) \times n_i + \left(S_{02} - \sum_{k=1}^{k=85} d_k^2 \times 12D_k\right) \times n_i + \left(S_{03} + S_{04} + \sum_{i=3}^{i=4}\sum_{k=1}^{k=85} O_i^k + \sum_{k=1}^{k=85} C_4^k\right) \times n_{i+} \sum_{i=5}^{m} S_{1i} \times n_i. \qquad (4)$$

Note. When assessing social inequality in the development of a mathematical model we examine the differences in the autonomous okrugs and oblasts in which they are included separately: for example, Khanty-Mansi Autonomous Okrug – YUGRA, Yamalo-Nenets Autonomous Okrug and the Tyumen Oblast without autonomous okrugs. Thus, the total number of subjects is defined as 85 units.

The mathematical model we developed demonstrates clearly that the proposals to improve the income tax system with the use of a flat tax scale and a system of tax deductions are aimed at simultaneously increasing the actual burden on citizens with incomes above average and reducing the burden for low-income citizens. This is the social condition of the income tax model; in order to substantiate its effectiveness it is necessary to conduct the second stage of the study – a statistical assessment of reliability of the sufficient condition.

4.2. Statistical verification of the social condition

The transition from the current to the proposed model of proportional income taxation is aimed at strengthening the social orientation of the tax by redistributing the tax burden among different groups of the population. The calculation of the social effect in income taxation is an important component of the state tax policy.

As we noted earlier, it is impossible to eliminate the existing income gap between the richest and the poorest citizens with the help of the tax scale. According to rough calculations, taking into account the fact that the R/P 10% ratio in Russia in 2017 was 15.3 and that its normative knowledge does not exceed 10, we conclude that the tax rate for the tenth decile under the simple progressive scale should be at least 45%, which carries significant economic, political and social risks.

In this regard, we consider only the lower five deciles as an estimate of social equalization, in which the stratification can indeed be reduced through the use of tax tools.

It is advisable to use per capita income after taxation broken down by decile groups as the indicators of alignment (indicators of the second stage of the system)

In accordance with the standard, the distribution of the population by decile groups should be as follows *(Tab. 2)*.

Table 2. Normative distribution of population by decile groups*

Indicator	Group number									
	1	2	3	4	5	6	7	8	9	10
Share of the population by decile groups, %	10	10	10	10	10	10	10	10	10	10
Share of income in each decile, %	a	2a	3a	4a	5a	6a	7a	8a	9a	10a
R/P 10% ratio (each decile to 1)	1	2	3	4	5	6	7	8	9	10

* Our findings based on the normative value of R/P 10% ratio (10 = 10 a/a); a – the share of the lower decile in the income structure.
Source: own compilation.

Based on the normative values presented above, at the second stage of verifying whether the model complies with the social condition, it is necessary to determine the compliance (non-compliance) of the actual distribution of average income after tax with its theoretical distribution. To do this, it is advisable to use Tukey's Q-test. It is applicable because the following conditions are true:

1) the size of the population by decile groups is the same: $n_1 = n_2 = ... n_m$;

2) it is possible to set the target values of average per capita income for five lower deciles.

Knowing the actual per capita cash income by decile groups x_1, x_2, x_3, x_4, x_5, we find out that the corresponding averages to be tested for equality, under the standard values of R/P 10% (1, 2, 3, 4, 5) will be: $5x_1$, $5/2x_2$, $5/3x_3$, $5/4x_4$, x_5.

The use of Tukey's Q-test to determine the equality or inequality of the averages is carried out in several stages:

1. Calculation of average income after tax $\bar{X}_1, \bar{X}_2 \ldots\ldots \bar{X}_m$ by the formula of arithmetic mean simple for each of the five decile groups.

2. The calculated per capita incomes are ranked in ascending order (they coincide with the decile sequence number) (5):

$$\bar{X}_1, \pi \bar{X}_2 \; \pi \bar{X}_3 \, \pi \ldots\ldots \pi \; \bar{X}_m \, . \qquad (5)$$

3. There are differences (of the first order) between adjacent average per capita incomes (6):

$$\bar{X}_2 - \bar{X}_1 \, ; \quad \bar{X}_3 - \bar{X}_2 \, . \qquad (6)$$

4. Similarly, we define the differences between average per capita incomes arranged in an ordered series skipping one decile (the difference of the second order), two deciles (third order) and three deciles (fourth order) (7):

$$\bar{X}_3 - \bar{X}_1 \, ; \quad \bar{X}_4 - \bar{X}_2 \, . \qquad (7)$$

5. For each value of the difference it is necessary to put forward two hypotheses: zero (H_0) – average per capita incomes by decile groups are equal (the goal of social alignment is achieved); alternative (H_A) – average per capita incomes by decile groups are not equal (the goal of social alignment is not achieved).

6. For each deviation in average per capita incomes (a pair of deciles) an average error (8) is calculated:

$$= \sqrt{\frac{S^2_{hab}}{n_m}} \;\; , \qquad (8)$$

where S^2_{hab} is the variance within each decile,

n – population size in each decile group.

7. For first-order differences, the actual value of the criterion is found by dividing them by the average sampling error (9):

$$Q_{fact\,(1)} = \frac{\bar{X}_2 - \bar{X}_1}{m}, \; Q_{fact\,(2)} = \frac{\bar{X}_3 - \bar{X}_2}{m} . \qquad (9)$$

The actual values of Tukey's Q-test are compared with the value presented in the Table (the same value for all the first order differences). The value in the Table depends on three characteristics: significance level (it is advisable to set it at the level of 95%), the number of degrees of freedom and the value k=2 for the first order differences. If $Q_{fact} > Q_{table}$, then the alternative hypothesis is accepted (the goal of social alignment is not achieved), otherwise the null hypothesis is accepted.

8. By analogy, the actual value of the criterion for the second, third, and fourth order differences is determined. The only difference is that the coefficient k equal to 3, 4 and 5 (respectively) is used to determine the table value of Tukey's Q-test.

In addition we note that, instead of substantiating the statistical hypotheses, it is possible to determine the elimination of social inequality in the lower deciles by comparing the pairwise differences in average income with the least significant difference (LSD). In this case, LSD is defined by formula 10:

$$\text{LSD} = Q_{\text{table}} \times m\ . \tag{10}$$

Where the actual differences are less than or equal to LSD, then excessive social inequality is recognized as eliminated. If the actual differences are greater than LSD, then the income tax model needs to be improved, because it does not meet the social condition.

4.3. Approbation of the mathematical model and verification of its conditions

According to the mathematical model we developed, we determined the volumes of revenues not received by the consolidated budgets of each of the constituent entities of the Russian Federation, which were subsequently used to comply with the budget conditions of the model. The methodology for calculating the revenues that were not received is based on the use of average characteristics, because positive and negative deviations are mutually compensated under this method of calculation.

The following assumptions were used in the assessment:

1. The subsistence minimum for the able-bodied population and for child rearing was used as of the 4th quarter of 2017.

2. The number of employed people with incomes below the subsistence minimum is adjusted to the average Russian indicator – "the share of the population of working age" (56%).

3. When calculating the indicators "Proportion of the population whose per capita income is within the range of one to two non-taxable minimums" and "Proportion of the population whose incomes do not exceed the average accrued monthly wage", the income boundaries are determined on the basis of statistical data on the distribution of the population by income size, taking into account correction factors. These factors are determined with the help of expert method, since the boundaries of the groups do not coincide exactly with the size of the subsistence minimum.

4. The number of individuals actually entitled to standard child rearing deductions is determined under the new rules, taking into account the coefficient of 0.25. The coefficient is determined on the basis of the average actual indicators of the Federal Tax Service statistical reporting in 2016 (every fourth individual receiving the income used child rearing tax deduction – Form No. 5-NDFL).

5. To determine the average number of children in the family in the constituent entity of the Russian Federation, the total fertility rate is used, which shows an average number of children born to one mother during the entire reproductive period (that is, from 15 to 50 years), while maintaining at each age the birth rate of the year for which the indicator is calculated. Its value does not depend on the age composition of the population and characterizes the average birth rate in a given calendar year.

6. The average number of months in which tax deductions were received under the current rules of the RF Tax Code is determined as the quotient of 350,000 rubles (the maximum amount of income for deduction) and the average monthly nominal accrued wage.

7. Statistical data of the Higher School of Economics are used to assess the structure of students (by sources of funding) in order to calculate the contingent of students[2]. In 2016, the share of students studying at the expense of budgets (federal, regional, local budgets)

[2] Gokhberg L.M., Kovaleva G.G., Kovaleva N.V. et al. *Education in Numbers: 2018: Concise Statistics Collection*. Moscow: NIU VShE, 2018. 80 p.

Table 3. Calculating the contingent of recipients of social deductions for education, 2016

Indicator	Average	Decile groups									
		1	2	3	4	5	6	7	8	9	10
Services in the system of higher education per 1 person, thousand rubles	67.7	3.3	11.1	24.9	37.3	86.0	77.2	102.4	123.3	110.6	101.2
Number of people in decile groups, thousand people	146.8	146.8	146.8	146.8	146.8	146.8	146.8	146.8	146.8	146.8	146.8
Expenditure on education in universities, thousand rubles	9943	48	163	365	548	1262	1134	1504	1810	1623	1486
Structure of expenditure on education broken down by decile groups, %	100.0	0.5	1.6	3.7	5.5	12.7	11.4	15.1	18.2	16.3	14.9
Source: own compilation based on Rosstat data.											

Table 4. Calculating the amount of shortfall in the revenues to be recovered

No.	Type of income	Sum, thousand rubles
1.	Total amount of shortfall in budget revenues caused by the provision of non-taxable minimum income, thousand rubles	591 268 463
2.	Amount of shortfall in tax revenues caused by the increase in standard child rearing deductions, thousand rubles	300 863 887
3.	Notional amount of tax deductions on the 1st, 2nd and subsequent children, thousand rubles	10 886 584
4.	Amount of additional budget revenues due to the imposed restrictions on social tax deductions, thousand rubles	9 509 508
5.	Total amount of additional shortfall in budget revenues, thousand rubles (1+2-3-4)	871 736 258
Source: own calculation.		

amounted to 47.4% of the total number of students. Based on this share, the number of students enrolled on a contract basis is determined.

When calculating the number of recipients of deductions for education, we use the average share of expenditures on education for 9–10 decile groups of the population by income. The indicator is determined on the basis of the data of sample surveys of household budgets[3] *(Tab. 3)*.

Thus, the share of tax deductions that were previously received by taxpayers of two higher decile groups (conditionally – with incomes above three average levels) is 31.2%.

Table 4 presents the resulting shortfall in budget revenues ($\sum SFR_{REF}$).

[3] www.gks.ru/wps/wcm/connect/rosstat_main/rosstat/ru/statistics/population/level/

The aggregate assessment for the Russian Federation has showed that if the changes we propose are implemented, then the amount of shortfall in the revenues of the consolidated budget of the Russian Federation will be about 871.7 billion rubles.

The research needs to be continued in order to determine how much the basic rate of individual income tax should be raised in order to compensate for the specified amount of the shortfall in revenues (ΔHC), that is, to perform the fiscal condition of the model (11):

$$\Delta\text{HC} = \frac{\sum SFR_{REF}}{\sum_{i=1}^{m} S_{0i} \times n_0} = \qquad (11)$$

$$= \frac{871\ 736\ 257\ 752}{31\ 751\ 561\ 775\ 600} \times 100 = 2.7 \text{ p. p.}$$

Since the calculation of the additional rate took into account only the average monthly nominal accrued wage, that is, labor income of

Table 5. Calculating transformed variables to determine Tukey's Q-test

Income group	Group's share in the total cash income of the population, %	Average per capita cash income, rubles per month	Average per capita cash income after the introduction of amendments to the legislation, rubles per month	Calculation of transformed variables	Transformed values of average per capita income for the purpose of calculating Tukey's Q-test, rubles per month
First	1.9	5 983	6 877 (x_1)	$5x_1$	34 385
Second	3.4	10 368	11 668 (x_2)	$5/2 \cdot x_2$	29 170
Third	4.5	13 704	15 004 (x_3)	$5/3 \cdot x_3$	25 007
Fourth	5.6	17 107	18 407 (x_4)	$5/4 \cdot x_4$	23 009
Fifth	6.8	20 875	22 175 (x_5)	$5/5 \cdot x_5$	22 175
Source: own calculations.					

Table 6. Actual and tabular values of Tukey's Q-test

Differences of mean values		Value of the difference, rubles	Value of Tukey's Q-test	
			Actual	Tabular
First order	x_1-x_2	5 215	2.069	3.261
	x_2-x_3	4 163	1.652	
	x_3-x_4	1 998	0.793	
	x_4-x_5	834	0.331	
Second order	x_1-x_3	9 378	3.722	4.041
	x_2-x_4	6 161	2.445	
	x_3-x_5	2 832	1.124	
Third order	x_1-x_4	11 376	4.514	4.529
	x_2-x_5	6 995	2.776	
Fourth order	x_1-x_5	12 210	4.845	4.886
Source: own calculation.				

citizens, we believe it is possible to increase the base rate of individual income tax by 2 p.p. rather than by 2.7 p.p. According to the data as of 2016, other income taxed at the base rate and declared by taxpayers amounted to five trillion rubles. Taking into account the average inflation rate (5.88% – 2018 to 2016), the taxes from the base of 5.3 trillion rubles will be additionally received.

Next, let us determine the feasibility of an additional increase in the tax rate to ensure the growth of revenues, taking into account the purchasing power of the population. To do this, we check the following inequation (12):

$$36709.2 \times 105.88 \times (1 - 0.13 - 0.02) \geq$$
$$\geq 1.5 \times 16087.92 \times 105.88 \qquad (12)$$
$$33037.55 \geq 25550.83\,.$$

With an increase in the base tax rate on individual income by 2 p.p. (up to 15%), it is possible to use our proposals in terms of tax deductions and non-taxable minimum while complying with the budget conditions (non-reduction of inflow of revenues to the budgets). However, for individual constituent entities of the Russian Federation the reduction of income is possible (due to the low level of remuneration), which is necessary to compensate for by providing grants for equalization of budgetary security.

Next, we assess the model with respect to the implementation of the social condition *(Tab. 5)*.

Let us calculate the average sampling error based on the residual variance of the feature, which is conditioned by random factors[4] (13).

$$m = \sqrt{\frac{S_{res}^2}{n}} = \sqrt{\frac{931442005166400}{146674541}} = 2520\,rub. \quad (13)$$

To prove the hypothesis, we calculate the actual values of Tukey's Q-test *(Tab. 6)*.

Zero hypotheses about the equality of averages for all pairs should be accepted, since the actual values of Tukey's Q-test are lower than the corresponding critical values. With a probability of error in five cases out of 100, it can be argued that the average values of income in terms of their optimal distribution do not differ significantly by decile groups. Consequently, the goal of income equalization in the lower deciles has been achieved.

5. Conclusions.

Scientific novelty of our research consists in the development of a new proportionate model for income taxation of individuals, the model contains elements of a hidden progression (non-taxable minimum, the improved system of individual income tax deductions).

Our approach to modeling based on the synergy of mathematical and statistical methods is of theoretical significance. The advantage of our methodology consists in the fact that, unlike the majority of existing approaches to assessing the impact on social inequality through the income ratio in extreme deciles, it makes it possible to estimate the degree of population stratification for each pair of deciles separately. In case of finding a strong degree of stratification only for individual deciles, the government has the opportunity to develop targeted tax incentives (or other individual incentive tools) for a certain group of citizens.

Moreover, the methodology we developed can be used in the absence of budget constraints. In this case, the mathematical model must be transformed by changing the right side of equation (4) to the target (expected) value of tax revenues from individual income tax (C_p). However, in such a situation, an expert assessment of expected revenues is required, which introduces an anthropogenic factor in the model that distorts the actual statistical assessment of income taxation parameters.

Practical significance of the results of our study lies in the fact that the Ministry of Finance of the Russian Federation can use the materials and generalizations contained in the research for the purposes of improving the system of income taxation of citizens. The Federal Tax Service can use our research for the purposes of tax planning and forecasting the amount of tax deductions; the Ministry of Economic Development of the Russian Federation – for assessing the cumulative financial implications and the shortfall in the revenues of budgets within the budget system of the Russian Federation.

The use of mathematical modeling as a tool to substantiate a fair system of income taxation is most reasonable from the point of view of assessing all the conditions laid down in the tax reform (mandatory, sufficient, necessary). However, it is not appropriate to use such models to determine the inflow of revenues from individual income tax for a long-term period. Separation from economic theory is the main disadvantage of modeling, since any socio-economic system in it is represented only by a mathematical expression. It does not take into account changes in the global environment, political changes, and changes in legislation.

[4] Due to the lack of actual data, we calculated residual variance on the basis of the data on average per capita income in constituent entities of the Russian Federation, weighted by the population for each of the entities.

As we noted earlier, taxes are not a panacea for social inequality; fiscal regulation alone is not enough to solve such an important national economic problem. In this aspect, the evolution of our study may consist in the creation of a whole system of equations, taking into account both tax and other types of legislation – labor, social, credit.

Taking into account that it is rather problematic to set all constraints in the form of a mathematical function (in contrast to the fiscal constraint used in our model), we find it advisable to develop a homogeneous system of differential equations, where the constraints will be formulated in the form of limits of some functions.

References

1. Andreev M.Yu., Pospelov I.G., Pospelova I.I., Khokhlov M.A. *Tekhnologiya modelirovaniya ekonomiki i model' sovremennoi ekonomiki Rossii* [Technology of Economy Modeling and Models of Russian Modern Economy]. Moscow: MIFI, 2007. 262 p.
2. Gordon R.H., Kopczuk W. The choice of the personal income tax base. *Journal of Public Economics*, 2017, vol. 118, pp. 97-110.
3. Lehmann E., Marical F., Rioux L. Labor income responds differently to income-tax and payroll-tax reforms. *Journal of Public Economics,* 2013, vol. 99, pp. 66-84.
4. Long X., Pelloni A. Factor income taxation in a horizontal innovation model. *Journal of Public Economics*, 2017, vol. 154, pp. 137-159.
5. Morini M., Pellegrino S. Personal income tax reforms: a genetic algorithm approach. *European Journal of Operational Research,* 2018, vol. 264, no. 3, pp. 994-1004.
6. Kovárnik J., Hamplová E. Value tax equity and tax literacy as causality of tax incidence. *International Journal of Economics and Statistics*, 2013, vol. 1, no. 2, pp. 68-76.
7. Hungerman D.M. Public goods, hidden income, and tax evasion: Some nonstandard results from the warm-glow model. *Journal of Development Economics*, 2014, vol. 109, pp. 188-202.
8. Petrucci A. Optimal income taxation in models with endogenous fertility. *Journal of Macroeconomics*, 2015, vol. 43, pp. 216-225.
9. Tran Ch. Temptation and taxation with elastic labor. *Economic Modelling*, 2018, vol. 70, pp. 351-369.
10. Ching-Chong L., Chih-Hsing L. Optimal nonlinear income taxation with productive government expenditure. *International Review of Economics & Finance*, 2012, vol. 22 (1), pp. 66-77.
11. Liskina E.Yu. Some mathematical models of taxes and dues in Russia. *Vestnik Ryazanskogo gosudarstvennogo universiteta im. S.A. Esenina=Bulletin of Ryazan State University named for S.A.Yesenin*, 2014, no. 2 (43), pp. 168-182. (In Russian).
12. Smirnov R.O., Chistyakov S.V. Modeling the choice of the regressive scale of the unified social tax. *Vestnik Sankt-Peterburgskogo universiteta. Ekonomika=St Petersburg University Journal of Economic Studies*, 2003, no. 4 (29), pp. 79-85. (In Russian).
13. Smirnov R.O. Modeling of choosing the parameters of the income tax schedule. *Vestnik Sankt-Peterburgskogo universiteta. Ekonomika=St Petersburg University Journal of Economic Studies,* 2011, no. 4, pp. 141-148. (In Russian).
14. Bertotti M.L., Modanese G. Micro to macro models for income distribution in the absence and in the presence of tax evasion. *Applied Mathematics and Computation*, 2014, vol. 244, pp. 836-846.
15. Sybiryanska Yu.V., Kondratenko M.B. The new model of the tax system of Ukraine: the redistribution of the tax burden. *Biznes inform,* 2014, no. 7, pp. 276-283.
16. Mel'nikov Yu.B., Onokhina E.A., Shitikov S.A. Improving the adequacy of economic models. *Izvestiya Ural'skogo gosudarstvennogo ekonomicheskogo universiteta=Journal of the Ural State University of Economics*, 2018, vol. 19, no. 1, pp. 94-106. (In Russian).

17. Mironov V.V., Smirnov A.V. On the problem of mathematical modeling of simulation models of socio-economic development of the region. *Korporativnoe upravlenie i innovatsionnoe razvitie ekonomiki Severa: Vestnik Nauchno-issledovatel'skogo tsentra korporativnogo prava, upravleniya i venchurnogo investirovaniya Syktyvkarskogo gosudarstvennogo universiteta=Korporativnoe upravlenie i innovacionnoe razvitie economiki Severa*, 2012, no. 3, p. 8. (In Russian).
18. Allanson P. Income stratification and between-group inequality. *Economics Letters*, 2014, vol. 124, no. 2, pp. 227-230.
19. Molero-Simarro R. Inequality in China revisited. The effect of functional distribution of income on urban top incomes, the urban-rural gap and the Gini index, 1978–2015. *China Economic Review*, 2017, vol. 42, pp. 101-117.
20. Khosravi Tanak A., Mohtashami Borzadaran G.R., Ahmadi J. Entropy maximization under the constraints on the generalized Gini index and its application in modeling income distributions. *Physica A: Statistical Mechanics and its Applications*, 2015, vol. 438, pp. 657-666.
21. Tikhonova A.V. Non-taxable minimum for individual income tax as a tool of socialization of the tax system of Russia. *Nalogi=Taxes*, 2018, no. 4, pp. 36-40. (In Russian).
22. Kalinina O. Economic-mathematical models of progressive income taxation in Russia. *International Journal of Economics and Statistics*, 2015, vol. 3, pp. 39-47.

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