UDC 338.27:330.42, LBC 65.23

The use of median rank for the comparison of alternatives in the long-term

Sergey L'vovich SADOV
Doctor of Economics, Leading Research Associate, Laboratory for Complex Fuel-and-Energy Issues, the Institute of Socio-Economic and Energy Problems of the North Komi Science Centre, Ural Branch of RAS (26, Kommunisticheskaya Street, Syktyvkar, 167982, Russia, sadov@energy.komisc.ru)

Abstract. The article offers the author's approach to the comparison of the alternatives for the long-term development of the system/object, based on the use of ordinal variables. The best alternative is compared and selected using the author's method of supramedian ranks. The article illustrates the application of the proposed approach when choosing the option of economic development of the Komi Republic.

Key words: alternatives for the long-term development of the system/object, supramedian rank method, ordinal variables.

It is necessary to forecast socio-economic processes for the long term, to assess the variants of their dynamics and to give recommendations on selecting the alterna-tive in the conditions of high uncertainty. This results from the large forecast hori-zon, and as a consequence, great uncer-tainty of the external conditions – the environment, in which the forecast object (system) develops and functions, and low reliability of the majority of the original data. Socio-economic systems, as a rule, are very complex, which leads to great and essentially inherent uncertainty in determining the dynamics of their development. Therefore, when modeling economic systems, it is typical that expert evaluations serve as an important part of the information support of the models.

Moreover, it is desirable to minimize specific quantitative evaluations, provided by the experts, that in the long-term are invariably inaccurate and can misinform the researcher. The aim of the author was to develop a method for selecting the best alternative for the development of the object/system, based on the population analysis of expert qualitative evaluations, which goes beyond grouping the alternatives by preference, and allows differentiating them to the maximum degree.

The means of the fuzzy sets theory were used previously, when dealing with the specific features of uncertainty in the economic evaluation of oil and gas resources, and have suited well to the modeling problems of geological objects and bearing areas and to the issues of estimating the economic value of hydrocarbon resources [1]. It remains unclear whether the situation is favorable for the long-term forecasting problems of socio-economic processes. For example, the construction of the membership function, one of the most important means of the fuzzy sets theory, in each specific task requires different approaches and consideration of particular characteristics of the industry. Therefore, it proved to be useful to address other possibilities for forecasting in conditions of uncertainty, provided by mathematics.

For these purposes, nominative variables, ordinal variables, interval variables and ratio variables are widely used along with the fuzzy variables. They are listed in the order of increasing informative value and the requirements to the initial information are increasing along in the same order. Ordinal variables are of special attention in the list. This is the type of a discrete variable of qualitative nature, used to indicate ordinal or rank indicators. They are expressed numerically in points or verbally in the form of a linguistic scale gradation (which can also be easily converted into quantitative indicators). They require less information support – above mentioned expert evaluations are sufficient. The examples of ordinal variables application in forecasting and management problems are mentioned in the works of T. Saati [2], Ya.I. Kurgin [3], on sociological, psychological and linguistic topics. There are no any serious obstacles for their application when solving the tasks of the economic forecasting of the qualitative character.

This approach is implemented in the well-known median rank method [4]. Although the user of the given method will have to regard a number of alternatives as equal and combine them into a single cluster, which is formally legitimate. But this is nonsense for the content of the task in hand: the alternatives for long-term development should provide *considerably different* ways of the development of the system under consideration, so one should not mix them, but to seek the best of them. The suggested supra-median rank (SR) method serves this purpose.

Let us consider the formal statement of the task of comparing several development alternatives of the object/system that can be associated with a region, sector or economic sector or private entity. The goal of the researcher is to determine the best alternative for the implementation, by means of comparison.

At the *first stage* (analytical) the expertise determines, on the one hand, the alternative – significant variants for the development of the object/system, which must exhaust the main possibilities and differ considerably from each other, and *m* factors, affecting the development of the object/system. The factors may be absolutely different in nature – economic, energy, social, investment, environmental, institutional, demographic, transport, etc. – the main point is that their effect is to be acknowledged as essential.

Further by each of the alternatives, each factor is conferred a rank, which will characterize its sufficiency, capacity to support and ensure the development of the object/system in compliance with the alternative, and the selection of the best alternative, based on the obtained set of ranks. Since the factors will be of different character and dimension, they should be reduced to a single scale of ranks, typically consisting of 5 or 9 levels. Let us consider the widely used scale, consisting of nine gradations [2]. For the problem under review:

- rank 1 is to define the excellent, highest capacity of the factor to ensure the considered development alternative;
- rank 3 –good capacity;
- rank 5 –satisfactory capacity;
- rank 7 –bad capacity;
- rank 9 – complete incapacity.

Even ranks from 2 to 8 express intermediate states. This establishes a link between quantitative values of ranks and their verbal description as a linguistic variable [5]. If the detection of even ranks for any factor will be very difficult, it will be necessary to apply the scale of 5 uneven levels. This will result in lesser "resolution capability" of the given factor, but will not change the method's principle. The analysis of the obtained matrix of factors will highlight the bottlenecks in the implementation of an alternative.

At the *second stage* (algorithmic) the *supra-median rank method* is used to evaluate the alternatives to conclude, which alternative is preferential. Obviously, the alternative is to be chosen, in case all its ranks are the best. But such situation is a rare exception. As a rule, when assessing the factors the situation is rather discordant. In this case, a set of supra-median ranks attributed to the alternative is highlighted out of the sum total of ranks. At that, the reason that there can not and should not be too many implementation drivers for this or that development scenario and that an absolute majority of the factors out of their total number will be sufficient enough, will play the decisive role.

Let us consider the best or equal to median ranks as *supra-median*. They should make up the absolute majority among the ranks involved in the assessment of alternatives, i.e. their number is not less than m', where

$$m' = \begin{cases} \dfrac{m}{2}+1, \; if \;\; m - even\, number \\ \dfrac{m+1}{2}, \; if \;\; m - uneven. \end{cases}$$

In other words, *supra-median ranks* are m' of the ranks in the beginning of the series, ranked from the best to the worst. Up to this moment, the described method is reminiscent of the median rank method [4]. But the further course of actions is significantly different.

As a rule, it turns out that at the first stage many alternatives have the same median rank (MR). In that case, in order to differentiate them in detail, it is necessary to apply other criteria, used in the method (*figure*): the number of supra-median ranks (NSR) is calculated for the alternatives with the same MR, and the higher the NSR of an alternative, the more preferential it is.

When required, providing that any alternatives have the same NSR, special attention is paid to the structure of supra-median ranks – the more high ranks among them, the better is the structure, which is revealed by such criterion as the sum of SR (SSR). The lower SSR of an alternative, the higher its evaluation as a result. At that, of special importance is such subtlety as the conditional character of addition operation, as in theory, ranks are not necessarily to be expressed in figures (though it may be more familiar and more comfortable). Letter symbols and any pictorial images are acceptable, as long as the order is defined for their set. In this case the introduction of the addition operation on the set of the symbols is possible theoretically, though it is a troublesome and pragmatically unreasonable activity. Therefore the addition at this stage of the algorithm is important only as the means of reflection, qualitative analysis and quantitative assessment of the structure of the supra-median ranks of the alternative.

Sequence of criteria in the supra-median rank method

Median rank (MR) →	Number of supra-median ranks (NSR) →	Sum of supra-median ranks (SSR) →	Sum of all ranks (SR)

Table 1. Example of determining the best alternative by SR method

Alternative	Factor							MR	NSR	SSR	SR
	1st	2nd	3rd	4th	5th	6th	7th				
Alternative-1	5	2	7	6	3	6	4	5			
Alternative-2	4	7	5	2	4	3	6	4	4	13	
Alternative-3	1	7	4	6	4	2	8	4	4	11	32
Alternative-4	3	6	2	4	2	7	5	4	4	11	29

The last attempt to differentiate the alternative, in case all its previous results are the same is to calculate the sum of *all* ranks (SR), i.e. considering the whole set of ranks, obtained for the alternative. And only if they are equal, the alternatives are to be acknowledged as equivalent and the final choice is to be made by other critera. Perhaps, one should turn to the source data and check, if the assignment of the initially conferred factors to the rank values is appropriate. The sequence of criteria, used when required (see figure), allows pointing to the application of the advanced concept of the median rank, its generalization. Hence, the name of the SR method comes.

Example. Assume that 4 alternatives for the development of the object/system are considered and 7 factors are selected according to the results of the analysis, the following source data are prepared, applying the scale of 9 gradations (*tab. 1*).

The process of determining the median rank by the example of the first alternative is as follows: as there are 7 factors, in compliance with the formula $m' = 4$. Among the set of ranks of the given alternative, we look for the highest one – rank 2 in this case. Can it be median? No, as it is the only one, and $1 < m'$. Further we consider whether the next rank is median – rank 3 in this case. Also no, as the number of the third and better ranks is $2 < m'$. Rank 4 together with the best ones also does not form the majority, therefore, can not be median. And only rank 5 is the median. Similarly the median ranks for the alternatives from the 2nd to the 4th are found and for all of them it makes up 4. If some alternative had the highest median rank, it would be recognized as the best and the work by the given method would end at this. But in this example, three alternatives have the highest MR (note that in compliance with the median rank method they should all be enrolled in one cluster), so the second criterion – the number of supra-median ranks is applied to this group. As 4 is the number for all the alternatives, the choice cannot be made, so the next criterion – the sum of supra-median ranks comes into action. This amount for "alternative 2" is equal 4+2+4+3=13, for the third: 1+4+4+2=11, for the fourth: 3+2+4+2=11. For the last two alternatives with the best values of the SSR criterion, they were equal, therefore the last criterion – the sum of the ranks, the value of which is 32 for "alternative 3", and 29 for "alternative 4", is to be applied. As the result of "alternative-4" is the best and the supra-median rank method ends at this.

Let us turn to the particular task of evaluating and selecting alternatives for the economic development of the Komi Republic in the long term. The following 4 alternatives (except the conservative-inertial alternative), promising in terms of socio-economic development of the region, can be singled out for the Republic [6, 7]: No.1 –the extension of raw-material orientation, No.2 – focus on the increase in the processing of raw materials with high added value, No.3 – the creation of new industries, No.4 – innovation. At least 6 factors will affect the possibilities of the alternatives' implementation: investment, technology, personnel, transport, institutional, environmental (that is the order they are included in *tab. 2*), therefore $m' = 4$.

Table 2. Factor matrix for selecting g the alternative for the economic development of the Komi Republic

Alternative	Factor						MR	NSR	SSR	SR
	1st	2nd	3rd	4th	5th	6th				
No. 1	6	2	3	7	4	8	6	–	–	–
No. 2	3	4	3	5	3	6	4	–	–	–
No. 3	4	5	5	5	5	5	5	–	–	–
No. 4	3	7	6	3	2	2	3	–	–	–

In order to assign ranks to the alternatives by any of the factors, it is necessary, first of all, to analyze how essential this factor is for implementing alternatives, assessing its impact, and then correlating its affect with a 9-grade scale of ranks.

Thus, for example, the investment factor is the most unfavorable for the alternative No. 1, as a part of the Timan-Pechora oil and gas province, located in the Komi Republic, is mainly comprised of deposits in the stage of declining production, and of prospective resources with problematic localization, therefore it is of the utmost interest to the major oil companies (having large investment opportunities). Whereas small companies do not have sufficient investment resources for prospecting and exploration and for developing the potential of the understudied areas of the province. The coal resources development is also very capital intensive.

The investment factor is less significant for alternative No. 3 and even lesser (approximately to the same extent) – for No. 2 and No. 4. Based on the correlations, the closest numerical gradations conformable to their linguistic values are selected on the 9-grade scale. As a result, the quantitative values of grades, equal to 6, 4, 3 and 3 respectively, were selected. The same is for other factors. The important development trend of the method described above is to make the process of assigning numerical values of ranks more objective through its unification and formalization.

Thus, factor matrix has the form (see tab. 2). It is noticeable that already at the first stage of the SR method the Alternative No.4 reveals itself as the most promising for implementation.

In conclusion, it should be noted that it is useful not to consider the SR method in isolation from economic assessments, even though they are approximate and obtained through long-term forecasting, in order to study the forecasts more thoroughly. Let us suppose that an Alternative (No.2, extension of raw-material orientation) is recognized as the most desirable for the successful socio-economic development of the Republic. Then it would be useful to compare the costs of the bottlenecks expansion of this alternative (in this case, 2nd and 4th factors that are to be raised to the rank 3) with the positive consequences of its implementation. In other words, it is necessary to determine whether it is worth to fight for this specific alternative or it will be better to focus on the one that is more consistent under current conditions. One of the ways to improve the given method may be to maintain accounting of nonequivalence factors, by which the practicability of alternatives is assessed – for the cases with certain prevalence of some factors within the whole forecast period.

References

1. Sadov S.L. *Metody otsenki neftegazovogo potentsiala territoriy* [Methods for Assessing Oil and Gas Potential of the Territories]. Syktyvkar: KNTs UrO RAN Publ., 2007. 248 p.
2. Saati T. *Prinyatie resheniy: metod analiza ierarkhiy* [Decision-Making: Analytic Hierarchy Process]. Moscow: Radio i svyaz', 1993. 280 p.

3. Khurgin Ya.I. *Problemy neopredelennosti v zadachakh nefti i gaza* [Problems of Uncertainty in Oil and Gas Issues]. Moscow-Izhevsk: Institut komp'yuternykh issledovaniy, 2004. 320 p.
4. Orlov A.I. *Ekonometrika* [Econometrics]. Moscow: "Ekzamen" Publ., 2002. 576 p.
5. Zadeh L. *Ponyatie lingvisticheskoy peremennoy i ego primenenie k prinyatiyu priblizhennykh resheniy* [The Concept of a Linguistic Variable and Its Application to Approximate Reasoning]. Moscow: Mir, 1976. 165 p.
6. *Energoekonomicheskoe prognozirovanie razvitiya regiona* [Energy and Economic Forecast of the Region's Development]. Executive Editor: RAS Corresponding Member V.N. Lazhentsev. Moscow: Nauka, 2008. 356 p.
7. *Innovatsionnoe razvitie toplivno-energeticheskikh sistem Severa* [Innovation Development of Fuel and Power Systems of the North]. Executive Editor: O.V. Buryi. Syktyvkar: KNTs UrO RAN Publ., 2011. 244 p.