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Forecasting of indicators of the region's socio-economic development

The article analyses the statistical data on the volumes of investment in different spheres of the socio-economic system of the Udmurt Republic in the 1996–2010 period. The values of production and human capitals are predicted for the next five years with regard to the forecasting of the dynamics of investment volume based on economic and mathematical models. The article resolves the task of modeling dynamics of the gross regional product in the Udmurt Republic by applying production function constructed on the basis of statistical methods of correlation and regression analysis, and the issue concerning the forecasting of the dynamics of production and human capitals.

Investment, production assets, human capital, gross regional product, forecasting.

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Introduction

In the modern world, economic forecasts are needed to determine priority directions of the development of economic systems, to assess mathematically the consequences of the planned economic decisions, when building optimal economic management strategies. In this regard, forecasting is one of the important tools used in the formation of social development strategy and tactics.

Production capital, human capital, and the gross regional product are the main macroeconomic indicators assessing the state of the regional economy.

Production capital (basic production assets – BPA) is understood as the material and technical basis of the production process, which is replenished with capital investments. It is also exposed to wear and tear, as it loses its properties due to various factors. In contemporary economy the compliance of production assets to the level of modern scientific and technological progress is the main criterion determining the necessity of production assets renewal. Thus, production assets require the investment, optimal from the standpoint of the current state of the economic system.

The authors use econometric methods to forecast the volume of investments in production capital, and the model of production capital to forecast the dynamics of the production capital [1].

Human capital is another important macroeconomic indicator, which represents a storage of knowledge, skills, experience, health and culture. The authors distinguish the following components of human capital: education capital, health capital, culture capital. Human capital, as well as production capital is exposed to replacement, its amortization, however, differs from material-technical resources. During the first years of human capital functioning, the economic value of the employee's storage of knowledge and abilities is not reduced, as is the case with physical capital, but on the contrary, increases due to his/her physical ageing and accumulation of practical experience. As a rule, the rate of physical and moral retirement of the knowledge accumulated and qualification start exceeding the values of the continuous accumulation of production experience by the end of the second decade of service. Only since this moment begins the process of human capital 'devaluation'.

The authors distinguish investments in the components of human capital: education, health care and culture. The volume of investments in human capital is forecast by applying econometric methods. The forecasting of the dynamics of the human capital is based on the human capital model [2].

Production functions, reflecting the effect of production factors on the output indicators of the economic system are used, when solving the issues concerning the forecasting of the economic dynamics. The authors consider production and human capitals as input factors, i.e. production factors, and the gross regional product as an output indicator of the economic system.

Gross regional product (GRP) is the main characteristic of the production results, which is used to evaluate the level of economic development, economic growth rate, the analysis of labour productivity.

The authors forecast the gross regional product based on the constructed production function.

Figure 1 shows the logical diagram of the study.

Thus, the authors conduct the forecasting of macroeconomic indicators in three stages: the investments in production and human capitals are forecast by econometric methods at the first stage; the values of production and human capitals are predicted based on mathematical models of these factors at the second stage; the third stage comprises the forecasting of the gross regional product with

Figure 1. Logical structure of the study

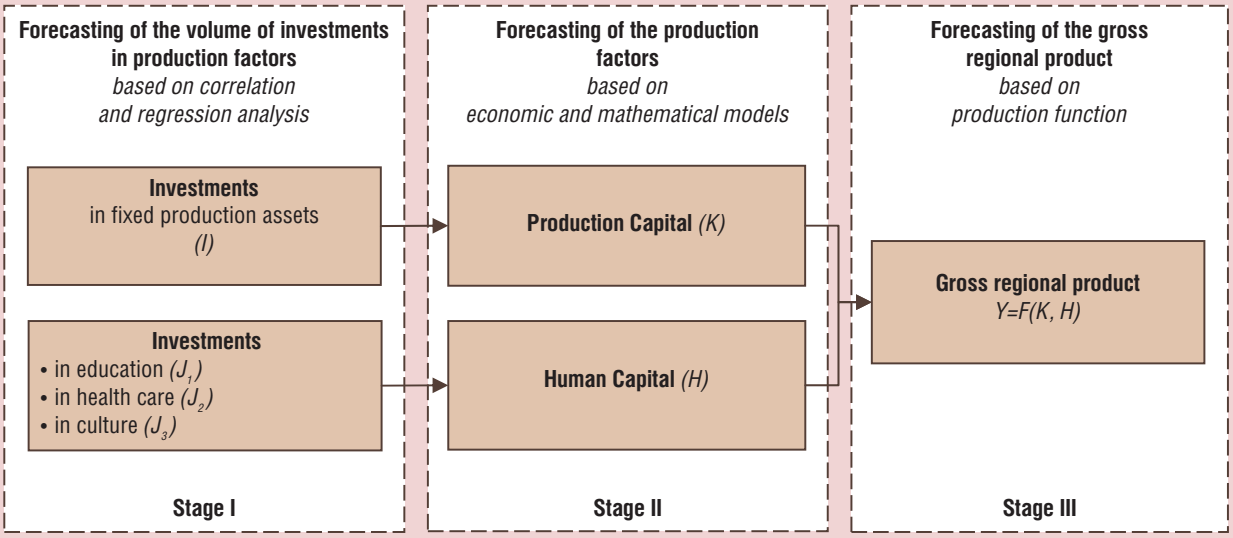
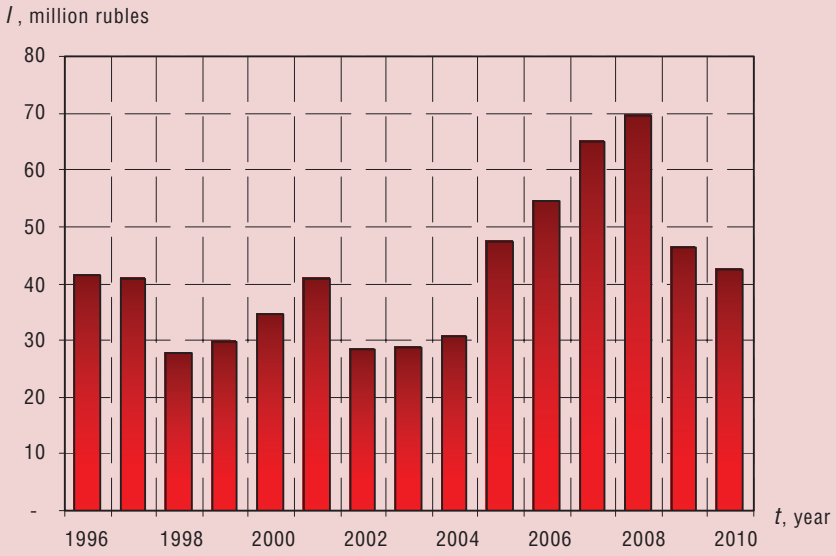


Figure 2. Dynamics of investments in production capital of the Udmurt Republic in the 1996 – 2010 period



regard to the value and dynamics of production and human capitals on the basis of constructed production function. The indicators of socio-economic development of the region are predicted in the case of the economic system of the Udmurt Republic (UR).

1. Analysis of the dynamics of the volume of investments in production factors

Investments in production capital (I) are the total expenditures on the creation and renewal of fixed production assets, comprising new construction, reconstruction and modernization of objects, the acquisition of machines, equipment, vehicles, etc.

The dynamics of investments in production capital of the Udmurt Republic in the 1996–2010 period [3] is shown in *fig. 2.*

The largest share of investments, involved in the region's production capital in 2008, amounted to 69 570 thousand rubles in 2010 prices, while the smallest volume fell on 1998 (27 870 thousand rubles). A high growth rate

of investments in production assets of the UR (24%) was observed in 2005–2008. A sharp decline in investments in production capital after 2008 can be attributed to the global crisis. In general, the average growth rate made up 2.8% in the 1996–2010 period.

Investments in human capital comprise investments in education (J1), health care (J2) and culture (J3). Investments in education contribute to the formation of highly qualified specialists, whose work has the largest impact on economic growth [4]. Investments in health care lead to the reduction of morbidity and mortality, prolonged working life of an individual [5]. Investments in culture reduce the level of social criminalization, increase the creative potential of an individual, shape moral values of a person, ultimately affecting the economic efficiency.

Statistical data on the investments in human capital of the Udmurt Republic in the 1996–2010 period [6] are shown in *figures 3, 4 and 5*.

The highest value of investment in education of the Udmurt Republic was observed in 2007 and amounted to 14.402 million rubles, while the least value fell on 1997 (5.457 million rubles). In general, there has been a tendency towards the indicator growth in the 1996 – 2010 period. The annual rate of the indicator growth for the period under review made up 6.4%.

The highest value of investment in the health care of the Udmurt Republic was observed in 2009 and amounted to 13.051 million rubles, while the least value fell on 1997 (4.447 million rubles). In general, there has been a tendency towards the indicator growth in the 1996–2010 period. The annual rate of the indicator growth for the period under review made up 9.5%.

The highest value of investment in culture of the Udmurt Republic was observed in 2008 and made up 2.780 million rubles, while the least value fell on 1997 (964 thousand rubles). In general, there has been a tendency towards the indicator growth in the 1996–2010 period. The annual rate of the indicator growth for the period under review amounted to 6.5%.

2. Forecasting of the volume of investments in production factors.

Investments in production factors pertain to economic indicators with a rather complicated structure. The values of these indicators

Figure 3. Dynamics of investments in education of the Udmurt Republic in the 1996 – 2010 period in 2010 prices

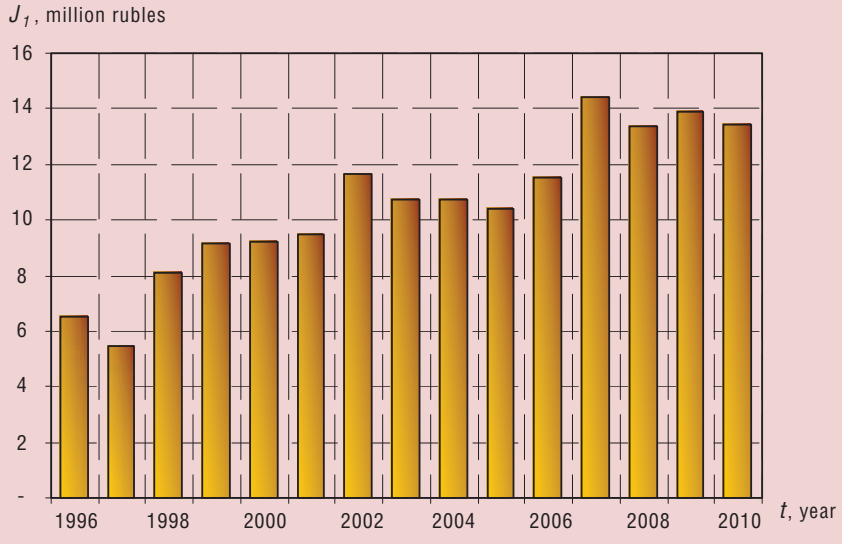


Figure 4. Dynamics of investments in the health care of the Udmurt Republic in 1996–2010

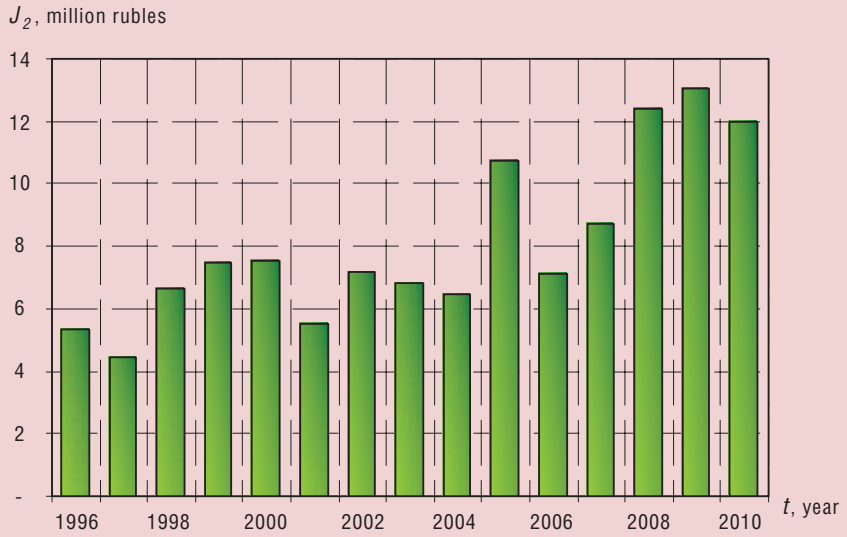
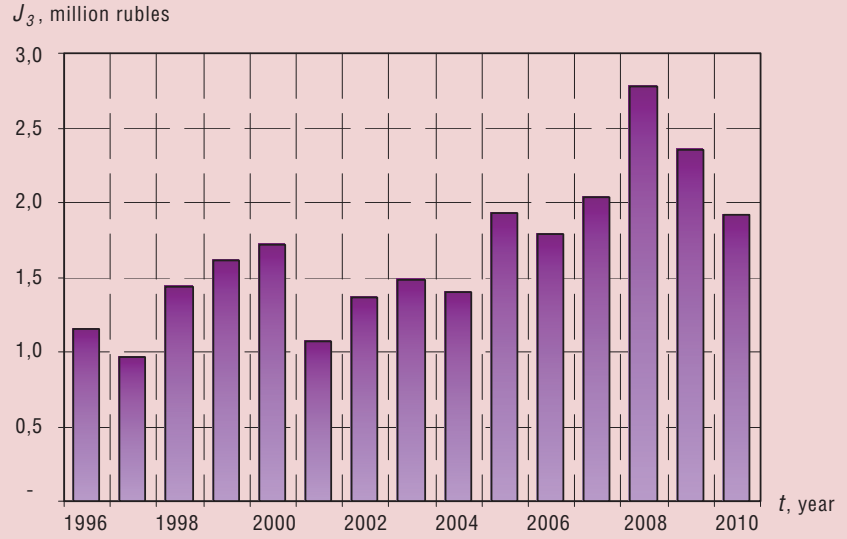


Figure 5. Dynamics of investments in culture of the Udmurt Republic in the 1996–2010 period



change over time in real life due to reasons and factors, that prevent constructing of a suitable classic multifactor econometric model, because of their multiplicity, measurement difficulty, insufficiency of theoretical assumptions concerning interrelations with the indicators. Therefore, the assumption is made with regard to the indicators of investments in production factors suggesting that internal regularities in the development dynamics are formed under the cumulative impact of various factors influencing the indicators, making it possible to apply an econometric model of the specific class of time series models.

Economic time series modeling by constructing the model of trend, seasonal and cyclical components does not lead to satisfactory results, while the residuals series frequently has statistical regularities. In such a case, autoregressive-moving-average models *ARMA* [7] are used to describe stationary time series.

But, as a rule, the economic indicators are nonstationary time series, therefore, autoregressive-integrated-moving average *ARIMA* (*p,q,k*) models are being used more widely. This model is also known as the Box-Jenkins model, a general view of which is represented by the formula:

$$\Delta^k y(t) = \mu_0 + \mu_1 \cdot y(t-1) + ... + \mu_p \cdot y(t-p) + \\ + \varepsilon(t) - \theta_1 \cdot \varepsilon(t-1) - ... - \theta_q \cdot \varepsilon(t-q), \quad (1)$$

where $y(t)$ – level value of the series at time t;

k – order (lag) difference;

μ_0, μ_1,…μ_p, θ_1, θ_q – estimated model parameters;

p – order of autoregressive process *AR*;

q – order of moving average *MA* process;

$\varepsilon(t)$ – error (white noise).

The order difference k in the model denotes a cyclic period and is determined on the basis of the sample autocorrelation function *AC* (k), which describes the correlation between the values of the studied process at different points in time. It is necessary to select such an order of difference, at which the value of the function *AC* (k) is significant [7].

In order to check the validity of the forecast models and the choice of the model forecasting short-term investments, let us calculate the following indicators:

– the determination coefficient:

$$R^2 = \frac{\sum_{t=1}^{T} (\tilde{y}_t - \bar{y})^2}{\sum_{t=1}^{T} (y_t - \bar{y})^2},$$

where $\tilde{y}_t$ – the series value under the calculated model;

– the Fisher statistic:

$$F = \frac{R^2}{1-R^2} \cdot \frac{T-m}{m-1},$$

where m – the number of estimated parameters in the model;

– the standard deviation:

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T} (\tilde{y}_t - y_t)^2}{T-m}},$$

– the Akaike criterion:

$$AIC = 2\frac{p+q}{T} + \ln\left(\frac{\sum_{t=1}^{T} (\tilde{y}_t - y_t)^2}{T}\right),$$

– the Schwarz criterion:

$$SHC = \frac{(p+q)\ln T}{T} + \ln\left(\frac{\sum_{t=1}^{T} (\tilde{y}_t - y_t)^2}{T}\right),$$

– the Durbin–Watson statistic:

$$DW = \frac{\sum_{t=2}^{T} ((\tilde{y}_t - y_t) - (\tilde{y}_{t-1} - y_{t-1}))^2}{\sum_{t=1}^{T} (\tilde{y}_t - y_t)^2}.$$

The characteristic roots of the autoregressive process and moving average process are used for verifying the stationarity and reversibility of the estimated models. The obtained roots should be of modulus <1 [7].

Let us consider the application of the Box-Jenkins model to forecast the volume of investments in production and human capital of the Udmurt Republic.

The model parameters *ARIMA* (*p,q,k*) for simulating the dynamics of investments in production capital will be estimated using sample and partial autocorrelation function *(tab. 1)*.

The coefficients of sample and partial autocorrelation functions of the first order are clearly significant, therefore, the following models have been chosen to simulate the dynamics of investments in production capital: *ARIMA* (1,0,0), *ARIMA* (0,1,0), *ARIMA* (1,1,0), *ARIMA* (1,0,1), *ARIMA* (0,1,1), *ARIMA* (1,1,1).

Table 1. Values of the sample autocorrelation function for investments in basic production assets

Function	Order, *k*						
	1	2	3	4	5	6	7
AC (*k*)	0.706	0.326	0.009	-0.136	-0.186	-0.188	-0.202
t-statistic *AC* (*k*)	3.861*	1.336	0.035	0.532	0.733	0.741	0.799
PAC (*k*)	0.706	-0.342	-0.132	0.045	-0.083	-0.064	-0.108
t-statistic *PAC* (*k*)	3.861*	1.410	0.516	0.174	0.323	0.248	0.421
* Significance at the 5% level.							

As follows from *table 2,* it is necessary to use the model *ARIMA* (1,0,0), the parameters of which are represented in *table 3,* to simulate the dynamics of investments in production capital.

Thus, the Box-Jenkins model describing the dynamics of investments in the production capital of the Udmurt Republic is as follows:

$$\tilde{I}(t) = 12384.9 + 0.71 \cdot I(t-1). \qquad (2)$$

with the determination coefficient R^2 of the model amounting to 0.72, and which has been used as the basis for forecasting investments in the production capital of the Udmurt Republic for the 2011–2015 period (see fig. 2)

The forecasting of investments in basic production funds by the model (2) demonstrates stable investment volume that will be observed up to 2015 inclusively (see tab. 7).

The parameters *ARIMA* (*p,q,k*) of the model for the forecasting of the volume of the investments in education $J_1(t)$, health care $J_2(t)$ and culture $J_3(t)$ are estimated similarly to the forecasting of investments in production capital.

The model *ARIMA* (2,2,0) is chosen to simulate the dynamics of investments in education *(tab. 4)*.

Thus, the Box-Jenkins model describing the dynamics of investments in the education of the Udmurt Republic ($R^2 = 0.82$) is the following:

$$\tilde{J}_1(t) = 18091.9 + 0.87 \cdot J_1(t-2) + \varepsilon(t) - 0.94 \cdot \varepsilon(t-2). \qquad (3)$$

The forecasting of investments in education by the Box-Jenkins model (3) assumes average growth of investments by 2.2%, which will be observed until 2015 (see tab. 7).

Let us estimate the parameters of the Box-Jenkins model to simulate the dynamics of investments in health care *(tab. 5)*.

The Box-Jenkins model forecasting the investments in the health care of the Udmurt Republic ($R^2 = 0.60$) is as follows:

$$\tilde{J}_2(t) = 15465.9 + 1.23 \cdot J_2(t-1) - 0.28 \cdot J_2(t-2) + \varepsilon(t) - 1.00 \cdot \varepsilon(t-1). \qquad (4)$$

The forecast values of investments in education according to the Box-Jenkins model (4) are represented in table 7. The average growth rate for the forecast period will make up 1.0% annually.

Let us estimate the Box-Jenkins model parameters for modeling the dynamics of investments in culture *(tab. 6)*.

The Box-Jenkins model forecasting the investments in the culture of the Udmurt Republic ($R^2 = 0.69$) is as follows:

$$\tilde{J}_3(t) = 2675.5 - 0.70 \cdot J_3(t-1) + 0.21 \cdot J_3(t-2) + \varepsilon(t) - 0.99 \cdot \varepsilon(t-2). \qquad (5)$$

The forecasting of investments in culture shows that the given indicator will grow up to 2015 by average 3.5% per year.

The results of the forecast of the volume of investments in production factors are represented in *table 7 and fig. 6*.

Table 2. Performance assessment of the models for forecasting investments in basic production assets

Model	R^2	F	σ	AIC	SHC	DW	Roots	
							AR	MA
ARIMA (1,0,0)	0.50	11.89*	13520.8	21.38	21.47	1.51	0.71	-
ARIMA (0,1,0)	0.45	10.49*	13029.3	21.39	21.49	1.54	-	-0.71
ARIMA (1,1,0)	0.54	6.41*	13520.8	21.44	21.58	1.92	0.56	-0.33
ARIMA (1,0,1)	0.02	0.23	10797.0	21.61	21.70	1.85	0.14	-
ARIMA (0,1,1)	0.02	0.30	103375.4	21.51	21.61	2.01	-	0.17
ARIMA (1,1,1)	0.58	6.89*	14122.0	20.93	21.06	2.00	1.49	2.29
* Significance at the 5% level.								

Table 3. Identification parameters of the model for forecasting investments in basic production funds

Coefficient	Value	*t*-statistics
μ_0	12384.9	1.38
μ_1	0.71	3.45*
* Significance at the 5% level.		

Table 4. Identification parameters of the model for forecasting investments in education

Coefficient	Value	*t*-statistics
μ_0	18091.9	7.78*
μ_2	0.87	7.33*
θ_2	-0.94	14.47*
* Significance at the 5% level.		

Table 5. Identification parameters of the model for forecasting investments in health care

Coefficient	Value	*t*-statistics
μ_0	15465.9	0.50
μ_1	1.23	2.89*
μ_2	-0.28	0.56
θ_2	-1.00	3.80*
* Significance at the 5% level.		

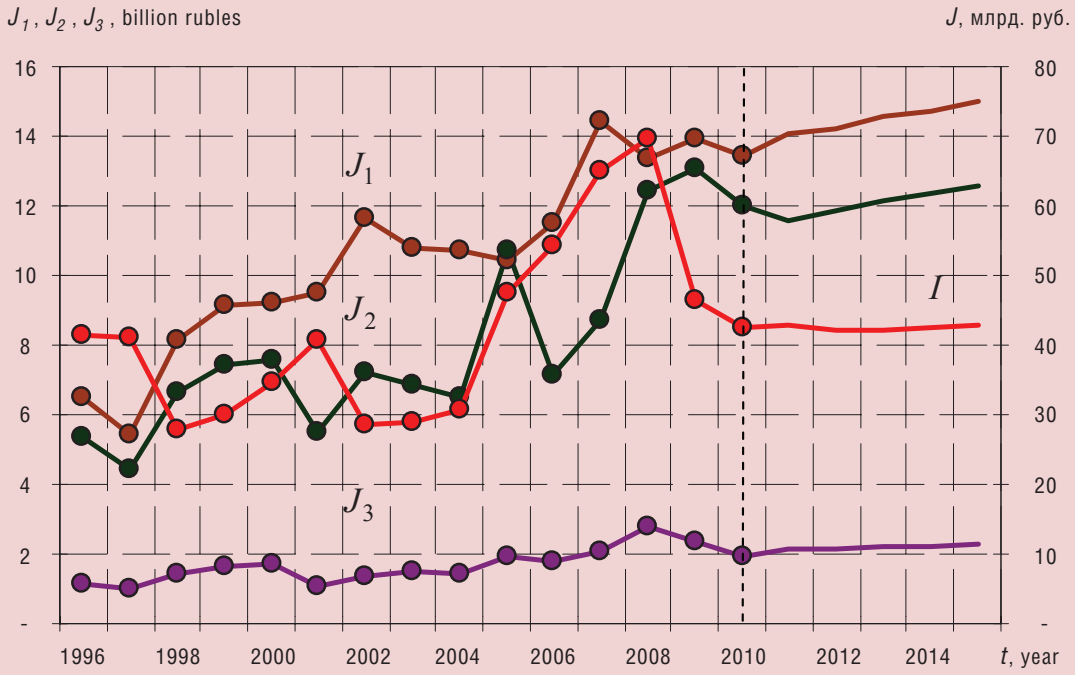
Table 6. Identification parameters of the model for forecasting investments in culture

Coefficient	Value	*t*-statistics
μ_0	2675.5	1.77**
μ_1	-0.70	2.48*
μ_2	0.21	0.74
θ_2	-0.99	3.91*
* Significance at the 5% level. ** Significance at the 10% level.		

Table 7. Summary table of the forecasts of investments in the production factors of the Udmurt Republic for 2011–2015 period, million rubles in 2010 prices

Year	Investments in production capital	Investments in education	Investments in health care	Investments in culture	Total investments in human capital
2011	42 772.04	14 057.52	11 603.93	2 124.80	27 786.25
2012	42 113.50	14 205.43	11 881.01	2 166.48	28 252.92
2013	42 012.77	14 576.43	12 138.21	2 205.00	28 919.65
2014	42 346.72	14 705.27	12 376.97	2 240.60	29 322.84
2015	42 933.72	15 028.44	12 598.59	2 273.51	29 900.54

Figure 6. Dynamics of investments in the production factors of the Udmurt Republic in the 1996–2010 and their forecast for the 2011–2015 period in 2010 prices



Having forecast the volumes of investment in production assets and human capital of the regional economy for the 2011–2015 period, it is possible to forecast the value of production and human capital for the same period.

3. Production factors forecast

The dynamics of the production capital of the Udmurt Republic is shown in *fig. 7*.

As follows from the chart, the basic production assets of the Udmurt Republic have been decreasing by 5.4% annually.

The following economic and mathematical model was used to simulate the dynamics of basic production assets [1]:

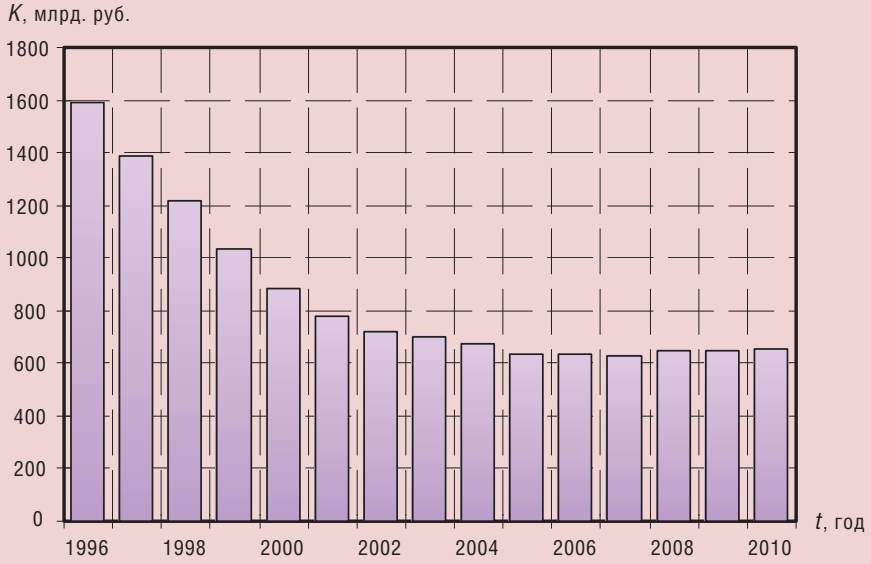
$$\frac{dK(t)}{dt} = I(t) - \eta K(t), \qquad (6)$$

with the initial conditions:

$$K(t = t_0) = K_0, \qquad (7)$$

where $K(t)$ – the amount of production assets at the specific time t; $I(t)$ – the amount of investments in production capital at the specific time t;

Figure 7. Dynamics of the production capital of the Udmuurt Republic in the 1996–2010 period in 2010 prices (smoothed data)



η – the retirement rate of production capital, determined as the weighted average value from the formula:

$$\eta = \frac{\sum_{t=1}^{T-1} K(t) \cdot (I(t) - \Delta K(t))}{\sum_{t=1}^{T-1} K^2(t)}, \qquad (8)$$

where $\Delta K(t) = K(t+1) - K(t)$.

The coefficient η for the Udmurt Republic makes up 0.116.

The forecast values of the production capital of the Udmurt Republic for the 2011–2015 period were obtained from the solution of the differential equation (6)–(7) using the numerical scheme of the explicit Euler method and with account of the forecast values of investments in basic production funds (see fig. 9, tab. 8).

The dynamics of the human capital of the Udmurt Republic is shown in *fig. 8*.

As follows from the chart, the human capital of the Udmurt Republic has been steadily increasing, amounting to 7.1% per year.

The economic and mathematical model of the human capital dynamics is represented in [2]:

$$\frac{dH(t)}{dt} = \bar{\varepsilon} J(t) - \chi H(t), \qquad (9)$$

$$H(t = t_0) = H_0, \qquad (10)$$

where $H(t)$ – the amount of human capital at the specific time t; $J(t)$ – the amount of investments in human capital at the specific time t; χ – human capital depreciation rate; $\bar{\varepsilon}$ – average share of population involved in production.

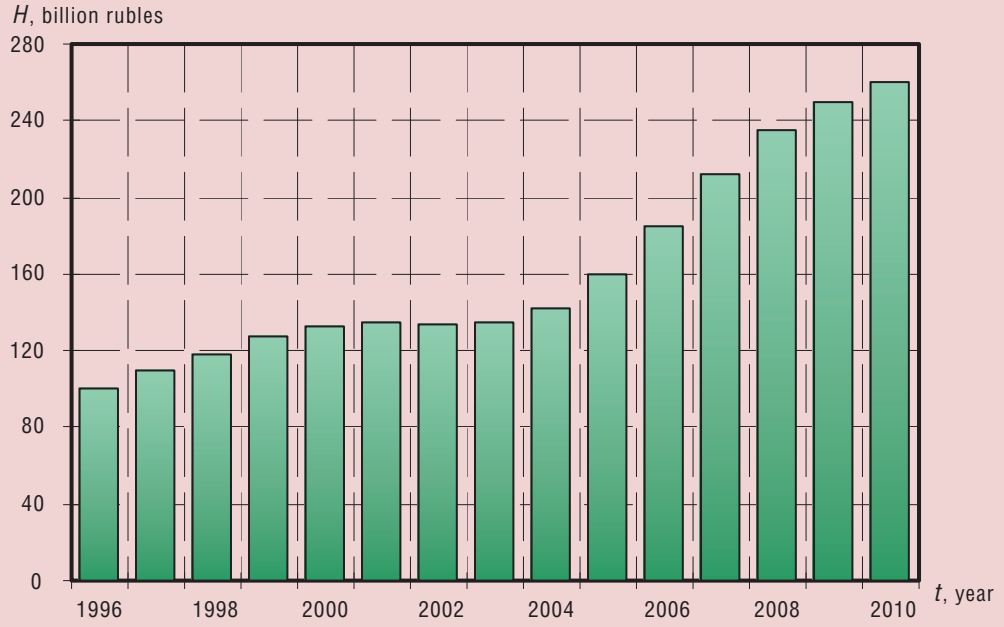
The parameters $\bar{\varepsilon}$ and χ have been estimated by the least square method according to the formula:

$$\begin{pmatrix} \bar{\varepsilon} \\ \chi \end{pmatrix} = \begin{pmatrix} \sum_t J^2(t) & -\sum_t J(t)H(t) \\ \sum_t J(t)H(t) & -\sum_t H^2(t) \end{pmatrix}^{-1} \cdot \begin{pmatrix} \sum_t J(t)\Delta H(t) \\ \sum_t H(t)\Delta H(t) \end{pmatrix} \qquad (11)$$

where $\Delta H(t) = H(t+1) - H(t)$.

Coefficients c and , calculated according to the statistics for the economic system of the Udmurt Republic make up 0.016 and 0.739 respectively.

Figure 8. Dynamics of human capital of the Udmurt Republic in the 1996–2010 period in 2010 prices (smoothed data)



The explicit Euler scheme has been also used for the numerical solution of differential equations (9)-(10). *Figure 9* represents the dynamics of the forecast values of the human capital of the Udmurt Republic for 2011–2015 years based on the forecast values of investments in human capital.

The production capital is expected to decrease (4.5% per year), while the human capital is forecast to increase (5.0 % per year) up to the year 2015 (see tab. 8).

Subsequently, understanding further development tendencies of the main production factors of the Udmurt Republic, it is possible to forecast the gross regional product.

4. Gross regional product forecast

Consider the production function in the form of the Cobb-Douglas function [8]:

$$Y(t) = A[K(t)]^{\alpha}[H(t)]^{\beta}. \qquad (12)$$

Let us restrict the linear homogeneity of the production function ($\alpha+\beta=1$), as the parameters of the evaluated function are statistically insignificant without considering the given condition due to high dependence (multicollinearity) between the input factors and *H*. Taking into account the conditions of homogeneity, the formula (12) is reduced to the form:

$$y_H(t) = A[k_H(t)]^{\alpha}, \qquad (13)$$

where $y_H(t)$=Y(*t*)/H(*t*), $k_H(t)$=K(*t*)/H(*t*).

The production function (13) was constructed on the basis of the smoothed statistical data on the values of human capital, production assets and gross regional product for the 1996–2010 period [6, 7].

The production function of the Udmurt Republic is the following (see fig. 4):

$$Y(t) = 0.84[K(t)]^{0.35}[H(t)]^{0.65}, \qquad (14)$$

with determination coefficient $R^2 = 0.84$, the Fisher statistic $F = 67.3$.

Figure 9. Dynamics of production and human capitals of the Udmurt Republic in 1996–2010 and their forecast for 2011–2015 in 2010 prices

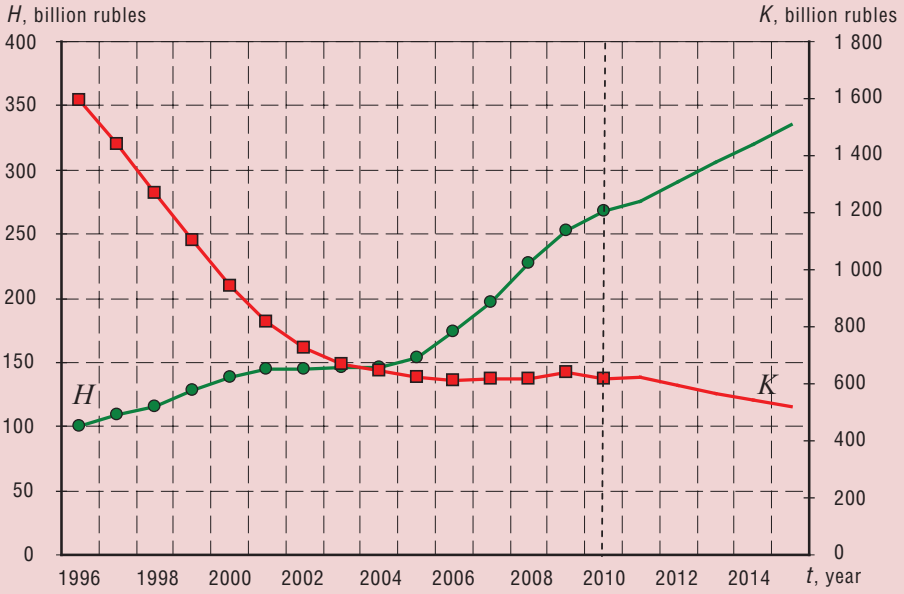


Figure 10 represents the graph of the production function of the Udmurt Republic in specific values. It should be noted that the model of the production function is close to the statistical values (k_H, y_H) across the Udmurt Republic.

The coefficient A = 0.84 shows that the level of technical progress for the region's economic system is low. The values of the elasticity coefficients of production capital ($\alpha = 0.35$) and human capital ($\beta = 0.65$) show that the increasing expenditures on production funds and human capital by 1% correspond to the rise in the production output by 0.35%, while the increasing expenditures on human capital by 1% comply with the increase in output by 0.65%. The elasticity coefficients ratio is $\alpha/\beta = 0.54$, hence, the economic system of the Udmurt Republic has been operating under the conditions of the second input factor (human capital) deficiency, extensive (fundreserving) growth.

When analyzing the production function, the authors consider the indicator, characterizing the possibility of replacing one factor with another, i.e. the marginal substitution rate of human capital with production funds [9]:

$$S_{HK} = \frac{\partial Y/\partial H}{\partial Y/\partial K} = \frac{\beta}{\alpha} k_H. \qquad (15)$$

The corresponding values (15) during the analyzed time-period are represented in *fig. 11*:

The value determines the increment ratio of the resources, separately resulting in the increment of one and the same production volume. Thus, for example, for the 2010, where $S_{HK} = 1.05$, it follows that in order to increase the production of one and the same number of Y, the factor K requires the increment 1.05 higher than the factor H. The obtained result, represented in fig. 10, indicates that the increase of investment in human capital is still more profitable ($S_{HK} > 1$).

The forecast dynamics of production and human capital gives the possibility to forecast the estimated production function (14) and the gross regional product of the Udmurt Republic *(fig. 12)*.

Figure 10. Graph of the production function and statistical data (k_H, y_H) of the Udmurt Republic

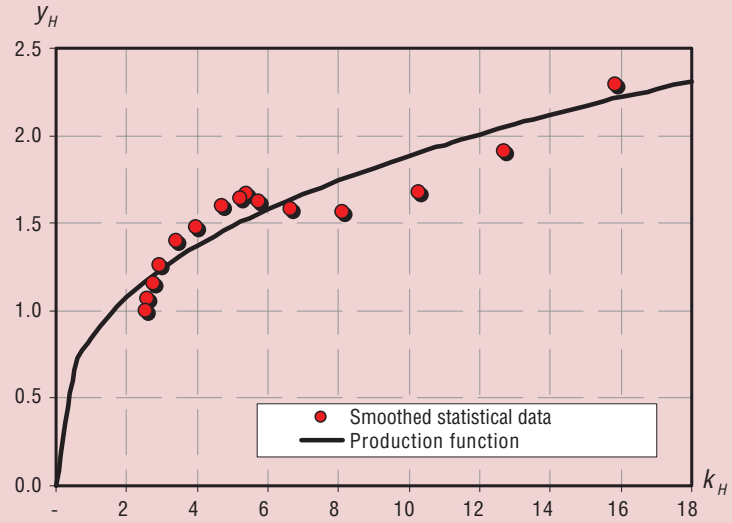
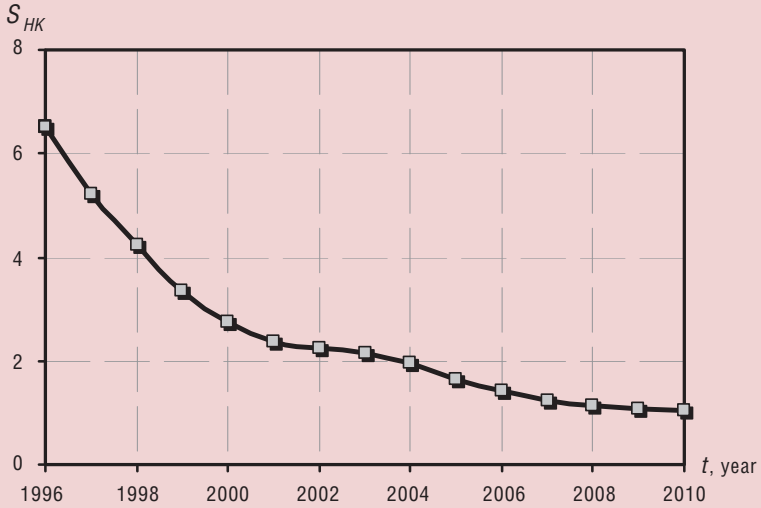


Figure 11. Marginal substitution rate of human capital with production capital in the 1996–2010 period



According to the conducted assessments, the GDP indicator is expected to grow up to the year 2015, due to the increasing growth rate of the region's human capital. The average growth rate of the GDP volume in 2011–2015 will reach 1.8%. In the most favourable economic conditions the forecast value of GDP in 2015 will amount to 330 926 million rubles, i.e. 9.0% higher than in 2010.

Conclusion

The results of the short-term forecasting of the main indicators of socio-economic development of the Udmurt Republic for the 2011 – 2015 period *(tab. 8)* have been obtained during the research, conducted according to the method, described in the work (see fig. 1).

Thus, we received the forecast values of the macroeconomic indicators of the Udmurt Republic for the 2011–2015 period, which have been obtained on the basis of the current trends in the development of the region's economic system. The expected further decrease in the volume of production assets will make on average 4.5% per year, the annual growth of

Figure 12. Dynamics of the Udmurt Republic GDP in specific (a) and basic (b) figures in the 1996–2010 period and its forecast for the 2011–2015 period in 2010 prices

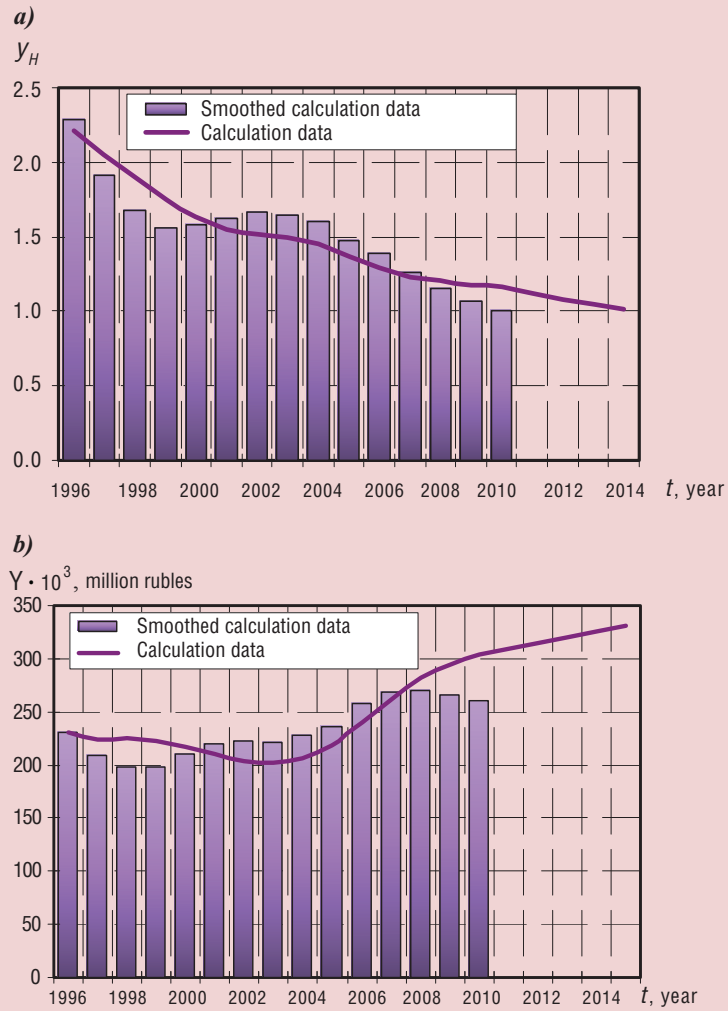


Table 8. Summary table of the forecasting of the Udmurt Republic macroeconomic indicators in 2011–2015

Year	Production capital			Human capital			Gross regional product		
	Forecast. million rubles	% to the previous year	% to 2010	Forecast. million rubles	% to the previous year	% to 2010	Forecast. million rubles	% to the previous year	% to 2010
2011	623 752.20	95.8	95.8	276 099.10	106.5	106.5	309 879.87	102.1	102.1
2012	593 660.53	95.2	91.2	290 775.58	105.3	112.2	315 045.34	101.7	103.8
2013	567 001.35	95.5	87.1	305 599.92	105.1	117.9	320 255.87	101.7	105.5
2014	543 393.60	95.8	83.5	320 578.31	104.9	123.7	325 542.29	101.7	107.3
2015	522 495.40	96.2	80.3	335 712.14	104.7	129.6	330 926.04	101.7	109.0

human capital will amount to an average of 5.0%. The gross regional product will increase by an average of 1.8% per year. It should be noted, that the forecast dynamics, obtained in the result of mathematical simulation, coincides with the moderately optimistic forecast of the

Udmurt Republic development up to 2015 [10]. The main condition of a moderately optimistic scenario development is the recovery of the region's economic sectors, with preserving the orientation to the exploitation of the available resources that include production and human capital.

The calculations showed that investments in human capital are more profitable, since the Udmurt Republic economy is currently under the conditions of the human capital deficiency, as follows from the production function of the region's economic system, obtained from the statistical data for the period 1996–2010.

The mathematical forecast method, presented in the paper, permits conducting parametric studies and analyzing ways to increase the growth rate of the region's macroeconomic indicators. Thus, for instance, it is necessary to implement one of the following scenarios, in order to increase the gross regional product by 5% annually in the 2011–2015 period:

To increase the volume of investment in production capital by 44.8% per year (i.e. 35 billion rubles per year) with maintaining the growth rate of investments in human capital. In this case, the current tendency towards the reduction of the production capital will change, and production capital will be annually increasing by 5.5% per year.

To increase the volume of government investments in human capital by 32.5% per year (i.e. 14 billion rubles per year) with maintaining the growth rate of investments in production capital. In this case, growth rates of human capital will rise from 5.0 to 9.9% per year.

To promote economic growth through the simultaneous increase of investments in production and human capital. In this case, to achieve an annual 5% growth of GRP, investments in production capital are to be increased by 20.8% per year (i.e. 12 billion rubles per year), and investments in human capital are to be increased by 18.8% per year (6 billion rubles per year). Besides, the volume of production assets will grow by 0.1% per year and the amount of human capital will rise by 7.8% per year.

Thus, in order to increase the current development rate of the socio-economic system of the Udmurt Republic, it is necessary to transfer the system to the new level of development, having upgraded the technological base in the priority sectors of the economy and having established an improved system of human resources capitalization.

References

1. Ketova K.V., Rusiak I.G. Identification and forecast of generalized indicators of regional economic system development. Applied econometrics. Moscow: Sinergia press, 2009. No. 3. P. 56-71.
2. Ketova K.V., Rusiak I.G. The economic and mathematical model of the analysis and forecast of the human capital factor. Economics, statistics, informatics. Vestnik Umo. Moscow: MESI, 2007. No. 2. P. 56-60.
3. Federal State Statistics Service of the Russian Federation. Available at: http://www.gks.ru
4. Galaeva Ye.V. Studies on human capital in foreign literature. Society and economics. 1997. No. 7. P. 244-255.
5. McConnell Campbell R. Economics: principles, problems, and policies. Kyiv: Khagar-Demos, 1993.
6. The report on the administration of the RF consolidated budget. Ministry of Finance of the Russian Federation; the Federal Treasury (Russia's Treasury). Available at: http://www.roskazna.ru
7. Ayvazyan S. A., Mkhitaryan V. S. Applied statistics and basics of econometrics. Moscow: UNITY, 1998.
8. Kleiner G.B. Production functions: theory, methods, application. Moscow: Finance and statistics, 1986.
9. Ashmanov S.A. Introduction to mathematical economics. Moscow: Nauka, 1984.
10. Economic news. Adopted forecast of socio-economic development of Udmurtia up to the year 2015. Svoyo Delo. Available at: http://svdelo.ru/ekonomika-udmurtii.