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Modeling of equilibrium operation of economy in the North-West federal district*

The article presents the results of economic equilibrium evaluating in the regions of North-west federal district from 2000 to 2008 made on the basis of the simple mathematical models reflecting the dynamics of consumers' demand, labour supply and usage of the labour potential in the market operation of the regional economy.

Regional economic equilibrium, labour activity, general equilibrium, local equilibrium, stability.

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It is important to study the formation and maintenance of local and general economic equilibrium in the market operation. This task is also interesting from a practical point of view. Economic and mathematical models of equilibrium operation of economy aren't the exact copy of the modeling object but they reflect its individual properties, which are important for the particular purpose and understanding of the certain aspects of reality. The examples of such models are the following: a behavior model of a homogeneous group "producers – consumers" [1, 4], a behavior model for the case of two clusters [3], a behavior model of m groups (m>1) relating to the same market of production and product's consumption with a restriction on the total number of "jobs" and without it [1, 2, 4].

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1. Model

This article presents the calculations performed by the model [4].

We are examining a single-component static model (hereinafter – the sequence of the related models). The main elements for the calculation of the model are "the members" – representatives of a homogeneous group. We considered the North-West federal district to be such groups and regarded employed and unemployed people (the economically active population of the region) as «the members» of that groups.

Assumptions of the model:

• The behavior of the market participants is a result of labor activity, when each group produces a product, and consumption of the released products. In addition, the group donates a part of the earned money to "social needs", taxes, savings, transfers, etc. Such payments are called "social burden" or "tax".

• The behavior of the group is efficient in the following case: if the regional charges are fixed, then considering the labour cost and consumption as unknown quantities, we can assume the relation between them as maximizing of an objective function, for example, the utility function.

The designations:

k – group number (region number);

N_k – group quantity;

ℓ_k – labour intensity of a certain member of the k-th group;

T_k – maximum labour intensity of a certain member of the k-th group (working capacity);

x_k – consumption volume of a certain member of the k-th group;

a_k – working efficiency per solitary labour intensity;

$a_k \cdot \ell_k$ – output of a certain member of the k-th group;

X^o_k – the minimum amount of the product that is necessary to produce for a certain group;

λ_k – a strictly positive parameter establishing the connection between individual income and its group quantity (it is called "attractiveness of the group" for convenience);

p – price of the product (it is formed by the common market);

q_k – cash charges ("taxes", "social burden").

We assume that the following budget constraint has been done in the market conditions: $p \cdot x_k = p \cdot a_k \cdot \ell_k - q_k$.

Utility function of a certain member of the k-th group is the following:

$$U_k(x_k, \ell_k) = \ln(x_k) + b_k \cdot \ln(T_k - \ell_k),$$

where b_k – a coefficient of individual measuring of the labour utility and consumption.

According to the mentioned budget constraint utility function is maximized at the point:

$$x_k = \frac{a_k \cdot T_k - \frac{q_k}{p}}{1 + b_k}, \quad \ell_k = \frac{a_k \cdot T_k + b_k \cdot \frac{q_k}{p}}{a_k \cdot (1 + b_k)} \cdot (1)$$

Two models of economic equilibrium are used in our research: local and general equilibrium.

Definition. *Local equilibrium of the k-th group is a such set of variables* $x_k^*, \ell_k^*, p^*, q_k^*$, where the following conditions are satisfied:

1) local balance:

$N_k \cdot (a_k \cdot \ell_k^* - x_k^*) = X^o_k$ (it's important that there is a fixed "load" for this group here);

2) hyperbolic distribution of income in the group:

$$N_k = \frac{\lambda_k}{p^* \cdot x_k^*};$$

3) maximizing the utility function of the group's members:

$$U(x_k^*, \ell_k^*) \geq U(x_k, \ell_k), \quad (2)$$

where $p^* \cdot x_k^* = p^* \cdot a_k \cdot \ell_k - q_k^*$.

It is clear (see [4]), that under the given parameters N_k, X^o_k, T_k, a_k, b_k, λ_k the local equilibrium exists and it is unique.

When all the groups (regions) are linked to the general market of production and consumption, it makes sense to talk about the general intergroup equilibrium.

Definition. *Intergroup general equilibrium* is the set of variables of system status $\{x_k^*, \ell_k^*\}$

and price parameters p^*, q_1^*, q_2^*, ..., $q_k^* > 0$, which satisfy the following conditions:

1) the total balance:

$$\sum_k N_k \cdot (a_k \cdot \ell_k^* - x_k^*) = X^0. \quad (3)$$

(only the general "load" is fixed here);

2) hyperbolic distribution of income in all the groups:

$$N_k = \frac{\lambda_k}{p^* \cdot x_k^*}; \quad (4)$$

3) maximization of the utility functions of participants of all the groups:

$U_k(x_k^*, \ell_k^*) \geq U_k(x_k, \ell_k)$ for all x_k, ℓ_k such, that

$p^* \cdot x_k^* \leq p^* \cdot a_k \cdot \ell_k^* - q_k^*$.

It is proved [4], that under the conditions

a) $\sum_k N_k \cdot a_k \cdot T_k > X^0$

b) $U_k(x_k, \ell_k) = \ln(x_k) + b_k \cdot \ln(T_k - \ell_k)$

the total intergroup equilibrium exists, it is unique and it can be found by the formulas:

$$p^* = \frac{\sum_k \lambda_k \cdot (1 + b_k)}{\sum_k a_k \cdot T_k \cdot N_k - X^0}, \quad (5)$$

$$q_k^* = a_k \cdot T_k \cdot p^* - \frac{\lambda_k \cdot (1 + b_k)}{N_k},$$

$$x_k^* = \frac{a_k \cdot T_k - \frac{q_k^*}{p^*}}{1 + b_k},$$

$$\ell_k^* = \frac{a_k \cdot T_k + b_k \cdot \frac{q_k^*}{p^*}}{a_k \cdot (1 + b_k)}.$$

The parameters of the general equilibrium form the local equilibrium for each group (region), and the equilibrium consumption and labour intensity $\{x_k^*, \ell_k^*\}$ will maximize the social welfare function $W = \sum_k \lambda_k \cdot U_k(x_k, \ell_k)$.

If at the certain time t equilibrium is disturbed, the value of "tax" q will change until it reaches the desired value for the given p. It is assumed that at each moment of time t price p and "tax" q form the following characteristics:

1) $x(p, q)$ – consumption of the individual participant,

2) $\ell(p, q)$ – labour intensity,

3) $n(p, q) = \frac{\lambda}{p \cdot x(p, q)}$ – labour supply in the amount of the current size of the group.

The mechanism of price and tax adjustment over the time while local equilibrium is being maintained can be described by the system of two differential equations:

$$\begin{cases} \frac{dp}{dt} = \mu \cdot \left(X^0 - \frac{q}{p} \cdot \frac{\lambda \cdot (1 + b)}{a \cdot T \cdot p - q} \right), \\ \frac{dq}{dt} = \nu \cdot \left(a \cdot T \cdot p - \frac{\lambda \cdot (1 + b)}{N} - q \right). \end{cases} \quad (6)$$

The condition of the stability of local equilibrium is the following inequality:

$$\frac{\mu}{\nu} > \frac{a \cdot T \cdot N \cdot X^0 - (X^0)^2}{\lambda \cdot (1 + b)}.$$

The process of maintaining of general equilibrium by a market mechanism is described by a system of $k + 1$ differential equations:

$$\begin{cases} \frac{dp}{dt} = \mu \cdot \left(X^0 - \sum_k \frac{\lambda_k \cdot (1 + b_k)}{a_k \cdot T_k \cdot p - q_k} \cdot \frac{q_k}{p} \right), \\ \frac{dq_1}{dt} = \nu_1 \cdot \left(a_1 \cdot T_1 \cdot p_1 - \frac{\lambda_1 \cdot (1 + b_1)}{N_1} - q_1 \right), \\ \ldots \\ \frac{dq_k}{dt} = \nu_k \cdot \left(a_k \cdot T_k \cdot p_k - \frac{\lambda_k \cdot (1 + b_k)}{N_k} - q_k \right). \end{cases} \quad (7)$$

It is more difficult to estimate the stability of general intergroup equilibrium but the stability of local equilibrium. This task can be solved by finding the constant characteristics of matrix of the partial derivatives from the functions represented in the right-hand parts of the system's differential equations (matrix size $(k + 1) \times (k + 1)$). In this case, negativity

of the real parts of all the $(k+1)$ eigenvalues of this matrix will be the condition of stability.

Thus using the above model on the real data will provide an opportunity to estimate not only the characteristics of economic equilibrium, but also the stability of general and local equilibrium and the possible changes in these indicators in dynamics.

2. Basic data

It is necessary to give the particular values to the parameters of described model to use it for the actual calculation. In consequence of it there is a problem of creation and justification of a particular "digitizing" procedure of the model's parameters.

To evaluate the economic equilibrium in the North-West federal district we used data on gross regional product, the final consumption of population, wages, employment and unemployment size represented on the official website of the Federal state statistics service, as well as materials of statistical compilations. At the same time we recalculated all the monetary figures at the prices in 2000 to give an opportunity to compare the annual data.

The procedure of "digitization" of the model's parameters consists of the following steps:

Step 1. Each region of the North-West federal district is given a number k. It could be a number in the list of subjects in the North-West federal district statistical yearbooks.

Step 2. We used the statistics of Federal state statistics service to find the values of variables Y – the gross regional product, X – the actual final consumption by the households, S – the wage fund, accrued to the workers on payroll and external by-workers, L – an average number of employees in the economy, B – the number of unemployed persons. These figures were taken from 2000 to 2008 in a comparable manner.

Step 3. We considered the group quantity to be equal to the volume of employees: $N_k = L_k$.

Step 4. We supposed that the labour intensity is reflected in remuneration of labour, so one employee's wage was calculated as:

$$\ell_k = \frac{S_k}{L_k}.$$

Step 5. Calculation of the maximum labour intensity:

$$T_k = \frac{S_k \cdot (L_k + B_k)}{(L_k)^2}.$$

Step 6. Calculation of consumption by a single participant:

$$x_k = \frac{X_k}{L_k}$$

(it is assumed that the entire volume of actual final consumption of the households falls on the persons employed in the economy).

Step 7. Calculation of minimum amount of the product that is required to be produced by the region: $X^o_k = Y_k - X_k$ (for local equilibrium; for general equilibrium

$$X^0 = \sum_k X^0{}_k).$$

Step 8. Calculation of labour productivity per unit labor intensity:

$$a_k = \frac{Y_k}{S_k}$$

(output per unit of payment).

Step 9. Calculation of the individual coefficient to measure the utility of labour and consumption:

$$b_k = \frac{Y_k \cdot B_k}{X_k \cdot L_k}.$$

This formula is derived from the assumption that the status of each region is optimal in the sense of maximizing its utility function.

Step 10. Calculation of the region attractiveness:

$$\lambda_k = \frac{X_k \cdot 10000}{\sum_k X_k}$$

(multiplier 10000 is used to zoom in). Since the general equilibrium is Pareto – optimal, there are coefficients to measure the usefulness of the regions and to point out the "attractiveness" to form the employees' quantities.

Thus, all the model parameters have got a specific meaning and numerical expression. Base data are presented in *appendix*.

Further, we consider in our calculation that the observed position of the regions of the North-west federal district forms a balance within the limits of the model described above.

3. The concept of analysis of the market stability according to the statistical reporting

Using the dynamical *equations (7)*, describing the process of "groping" of the stationary equilibrium, for the empirical analysis faces the methodological and procedural difficulties.

Firstly, how can we interpret the equilibrium prices p^*, defining the dependence of "taxes" q^* on them?

Secondly, how can we approach the empirical evaluation of "speed" of the market reaction μ and ν?

Thirdly, how can we describe this evolution of stationary equilibrium?

We will answer these questions when we describe the process of the development of regional economy in the terms of a single-product model.

Let's assume that all the model parameters *(a,b,T,...)* are changed exogenously. "Creeping" time varies discretely: 2000, 2001, ..., 2008. The system of regions is located in the economic equilibrium at such a moment. This balance is achieved by the process *(7)* during the "fast" time, when the system goes into a new equilibrium *p*(t)*, *q*(t)* from the state determined by the parameters *a(t)*, *b(t)*, *T(t)* ..., but estimated in the prices of the previous equilibrium *p*(t- 1)*, *q*(t-1)*. This transition is occurred with the "speed" μ and ν. We had denoted the right sides of the system *(7)* through *F(p,q)* and *Gk(p,q)* (here: q was a set of "tax" –transfers), so we got:

$$\Delta p = \mu \cdot \Delta F,$$

$$\Delta q = \nu \cdot \Delta G.$$

We can calculate μ and ν using these correlations.

As for the intentional sense of the equilibrium price p^*, we can note that its value is proportional to all values λ (see formula (4)).

Therefore, we can choose the suitable scale. When we divide λ into p^* we'll obtain the equilibrium price which is equal to 1. It means that all statistics are measured in equilibrium prices. Moreover, our conclusion about the stability remains valid.

4. The results of calculations

Evaluation of equilibrium prices and "tax" has shown that in the period from 2000 to 2008 the market operation of the North-West federal district was characterized by price decreasing and decline of «social duty» in all regions of the county *(tab. 1)*. Thus, the "tax" has decreased in 8.7 times in St. Petersburg for 9 years, also the price of national output has dropped by 2.4 times.

The average figures show that employees in the Vologda region were liable to the greatest "tax" and people in Pskov were liable to the smallest tax. In this case, we can explain negative values of "social duty" in Pskov region due to the fact that this region is subsidized.

Since the equilibrium maximizes social utility

$$W = \sum_k \lambda_k \cdot U_k(x_k, \ell_k),$$

then the value of the equilibrium price shows the value of the Lagrange multiplier to the appropriate task of maximization. It reflects the relative value of a produced unit and the value of the products which are consumed in the North-West federal district from the social utility point of view. The fall of this price over time implies the appropriate value decreasing that reflects the percentage of discounting. This process is accompanied by the growing scale of production and consumption in the regions. "Tax" q (loading price) is like a charge of each employee for participation in the economic life of the region, a single jobsite worth. In the market process (described by the proper differential equations), this cost depends on the correlation of the number of applicants to work (N_k) with the number of jobs (L_k).

Table 1. The dynamics of product price *(p)* и "tax" *(q)*

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Price (p)									
North-West federal district	0.029	0.026	0.024	0.022	0.019	0.017	0.015	0.013	0.012
"Tax" (q)									
Republic of Karelia	1.051	0.915	0.779	0.685	0.520	0.457	0.310	0.383	0.201
Komi Republic	1.948	1.539	1.109	1.106	0.874	0.701	0.646	0.487	0.478
Arkhangelsk oblast	1.646	1.384	1.171	1.202	1.424	1.282	1.142	1.186	0.975
Vologda oblast	2.020	1.732	1.53	1.531	1.511	1.296	1.106	1.030	0.828
Kaliningrad oblast	0.199	0.298	0.219	0.285	0.309	0.149	0.079	0.125	0.077
Leningrad oblast	1.212	1.103	1.214	1.391	1.173	0.980	0.940	0.875	0.854
Murmansk oblast	1.740	1.547	1.248	1.109	0.946	0.704	0.553	0.453	0.289
Novgorod oblast	0.612	0.697	0.488	0.514	0.461	0.349	0.230	0.167	0.089
Pskov oblast	0.296	0.171	-0.013	-0.14	-0.223	-0.271	-0.355	-0.257	-0.342
St. Petersburg	0.628	0.463	0.562	0.584	0.381	0.148	0.076	0.092	0.072
Source: Authors' calculations.									

Reduction of the «tax» was accompanied by increasing of marginal labour intensity (labour potential) in all the regions. The working potential of the market participants of Vologda oblast has increased in 2,1 times during the period from 2000 to 2008. While the Vologda region was only 6 among the regions of the North-West federal district by the average labour intensity. The Komi Republic showed the largest labour potential and Pskov Oblast showed the lowest labour potential *(tab. 2)*.

Assessing of the economic attractiveness of regions of the North-West federal district showed that during the entire period from 2000 to 2008 St. Petersburg was the most attractive for market participants: the values of the relevant parameter λ were higher in 5 – 12 times than the parameters of other regions *(tab. 3)*. Also during this period the attractiveness of St. Petersburg has increased by 15%. The same increasing of the attractiveness is typical for the Leningrad region. Other regions of the North-West federal district is not distinguished by their high values λ but they have lowered their economic attractiveness. For example, Murmansk oblast has lowered its economic attractiveness by 27%. Novgorod oblast is the least attractive.

During this period we observed the slowdown in the labour productivity (it was understood as the amount of output per unit of wage) per unit of intensity in all the regions of the North-West federal district *(tab. 4)*. The highest productivity per unit of intensity is fixed in Leningrad oblast. Pskov oblast has the worst productivity.

Table 2. Labour intensity ("labour potential")

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	29.597	32.114	37.456	38.923	42.397	48.904	51.558	59.581	60.973
Komi Republic	44.609	52.564	54.814	58.361	61.303	66.777	74.184	79.593	77.769
Arkhangelsk oblast	30.136	33.228	37.353	42.079	46.878	51.343	54.498	60.269	60.341
Vologda oblast	27.466	32.026	34.755	37.367	44.772	49.393	53.534	58.199	58.643
Kaliningrad oblast	18.020	20.434	24.991	27.525	27.352	29.235	36.371	42.118	49.374
Leningrad oblast	22.694	26.492	31.575	33.84	35.558	39.649	41.716	47.838	45.152
Murmansk oblast	43.14	48.885	52.261	52.324	54.427	58.439	63.873	71.485	72.055
Novgorod oblast	17.773	20.742	24.968	27.694	30.931	34.532	40.469	45.276	45.808
Pskov oblast	15.167	17.853	21.611	24.657	25.062	28.826	32.149	37.288	35.401
St. Petersburg	26.687	31.121	39.145	41.368	45.097	49.871	58.239	69.741	67.379
Source: Authors' calculations.									

Table 3. Region attractiveness (λ)

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	467.605	451.057	468.723	462.371	450.196	421.711	432.363	400.418	398.196
Komi Republic	849.596	965.564	948.777	913.128	899.835	840.159	822.57	793.539	744.278
Arkhangelsk oblast	835.757	882.717	855.003	865.259	838.717	813.969	799.85	804.17	813.274
Vologda oblast	773.161	770.186	751.059	732.356	700.727	657.989	665.541	652.188	632.684
Kaliningrad oblast	601.542	508.561	530.351	518.954	492.59	492.548	534.982	565.512	563.337
Leningrad oblast	782.258	802.804	792.929	783.206	840.175	879.032	879.834	873.931	840.966
Murmansk oblast	865.074	798.639	750.482	745.121	696.034	669.005	648.804	632.692	634.958
Novgorod oblast	420.719	396.402	391.669	370.269	348.755	330.752	334.646	338.293	359.138
Pskov oblast	376.84	369.127	405.94	435.519	436.918	402.293	408.32	364.535	377.161
St. Petersburg	4027.0	4055.0	4105.0	4174.0	4296.0	4493.0	4473.0	4575.0	4636.0
Source: Authors' calculations.									

Table 4. Labour productivity per unit of intensity *(a)*

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	3.127	2.927	2.640	2.514	2.362	2.237	2.097	2.016	1.938
Komi Republic	3.315	3.044	2.695	2.680	2.657	2.514	2.478	2.242	2.316
Arkhangelsk oblast	3.973	3.605	3.199	3.124	3.359	3.275	3.224	3.247	3.323
Vologda oblast	4.404	3.869	3.547	3.426	3.288	3.057	2.936	2.788	2.737
Kaliningrad oblast	3.711	3.149	2.759	2.695	2.893	2.764	2.402	2.402	2.272
Leningrad oblast	3.873	3.479	3.366	3.629	3.639	3.584	3.690	3.327	3.824
Murmansk oblast	3.450	3.064	2.708	2.697	2.671	2.498	2.295	2.096	2.094
Novgorod oblast	4.035	3.846	3.142	2.909	2.789	2.594	2.321	2.160	2.282
Pskov oblast	3.640	3.086	2.643	2.371	2.330	2.068	1.970	1.751	1.948
St. Petersburg	3.149	2.747	2.538	2.627	2.54	2.46	2.271	2.114	2.392
Source: Authors' calculations.									

Evaluation of the stability of the total inter-group and local equilibrium showed that the general and almost all local markets were not resistant in 2001, 2004, 2005, 2006 and 2007 *(tab. 5)*. It is noteworthy that there wasn't sustainable general equilibrium in the pre-crisis years (from 2004 to 2007). Perhaps the prolonged instability (in this case – the 4th year) is a specific economic behavior of the regions in the pre-crisis period, i.e. it serves as a harbinger of the crisis and an indicator of "unhealthy" development of the regional economy.

Of course, the instability cannot be interpreted unambiguously: it can reflect the condition of the Russian Federation in the whole, because each region is a part of it, and it also can show the specifics of a particular region. In any case, the instability means that the region cannot successfully cope with its economic problems. Since the instability of the common regional market of the North-West federal district indicates that the district cannot operate in isolation from others parts. At the same time the stability of the local market in Pskov oblast is not good housekeeping but it is a result of equilibrium maintenance through the mechanism of subsidies.

The greatest number of stable local equilibriums was noted in 2008. The local market of Leningrad oblast differed among all the local region markets in the North-West federal district by its instability: the economic balance in this region was not satisfying the criterion of stability during the whole period from 2001 to 2008. Local equilibrium in Vologda oblast was stable only in 2008 (see tab. 5).

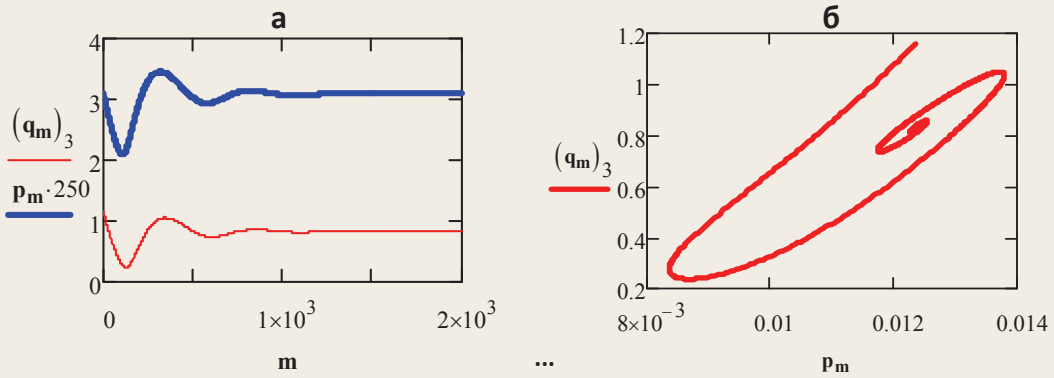
Stability of the general inter-groups balance in 2008 and the largest number of stable local

Table 5. Stability of general and local equilibrium

Region	Year							
	2001	2002	2003	2004	2005	2006	2007	2008
General equilibrium								
North-West federal district	–	+	+	–	–	–	–	+
Local equilibrium								
Republic of Karelia	–	–	–	–	–	–	–	+
Komi Republic	+	+	–	–	–	–	–	–
Arkhangelsk oblast	–	–	–	–	–	–	–	+
Vologda oblast	–	–	–	–	–	–	–	+
Kaliningrad oblast	–	–	–	–	+	–	–	+
Leningrad oblast	–	–	–	–	–	–	–	–
Murmansk oblast	–	+	–	–	–	–	–	+
Novgorod oblast	–	+	–	–	–	–	–	+
Pskov oblast	+	+	+	+	+	+	+	+
St. Petersburg	+	–	–	+	+	–	–	–

Source: Authors' calculations. Note: plus is a stable equilibrium.

Price and tax adjustment in the Vologda oblast during the maintenance of the general equilibrium in 2008



equilibriums can prove the crisis optimizing economic behavior of the regions of the District and increasing of its behavior rationality in crisis.

The following figure reflects the mechanism of price and tax adjustment in Vologda Oblast during the process of maintaining of the general equilibrium in 2008. As the market is stable, at the deviation from the equilibrium values the price and money transfers do not go away from them but tend to return to them (*fig. a* (a damped wave) – price schedules and tax rates depending on the time; *fig. b* (a spiral) – a tax schedule depending on the price).

Thus, modeling of equilibrium operation of economy in the North-West federal district through the application of mathematical behavior models of the homogeneous groups "producers – consumers" showed that the main trends of the regional labour market are:

- reduction of the total product's price;
- decline of "public burden" on the employed population ("tax");
- increasing of the marginal labour intensity, indicating the increasing of labour potential;
- decline in labour productivity per unit intensity;
- reducing of the economic attractiveness of the most regions in the North-West federal district with its increasing in St. Petersburg.

In this case, equilibrium was mostly unstable.

The above processes will inevitably affect the changing of labour supply and labour migration. They will make people employed in economics to work with the certain intensity, improve it and use the full labour potential.

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Appendix

Gross regional product (Y), value of the index for the year in 2000 prices, million rubles

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	28214,6	29004,6	31180,0	31678,8	32629,2	34880,6	36659,5	39775,6	37945,9
Komi Republic	59473,1	64171,5	61989,6	64407,2	67692,0	70399,7	76454,1	76148,2	78661,1
Arkhangelsk oblast	61806,9	65391,7	66438,0	71885,9	87269,5	95123,7	101972,6	114413,3	114413,3
Vologda oblast	69195,5	70510,2	72273,0	75597,5	82854,9	86583,4	90739,4	95367,1	92220,0
Kaliningrad oblast	23290,3	24035,6	26319,0	28766,6	32391,2	33557,3	38691,6	46391,2	48571,6
Leningrad oblast	56001,9	60706,1	70540,4	80627,7	87642,3	96056,0	106814,3	113543,6	119561,4
Murmansk oblast	55135,0	56017,2	54896,8	55500,7	57609,7	58992,3	60585,1	61918,0	61794,2
Novgorod oblast	20965,5	23523,3	23099,9	23908,4	25462,4	26430,0	27487,2	28999,0	31376,9
Pskov oblast	16178,9	16211,3	17021,8	17532,5	18619,5	18675,3	19590,4	20687,5	21328,8
St. Petersburg	188243,0	196902,2	231360,1	251025,7	268848,5	291162,9	315329,4	356637,6	389804,9

Source: authors' calculations according to the site of the Federal state statistics service and the collection of "Russia's regions 2009", used to convert the volume index of the GRP.

Actual final consumption of households (X), value of the index for the year in 2000 prices, million rubles

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	15931,6	17197,1	19858,5	20799,4	23200,9	25306,4	29259,7	29729,7	32247,4
Komi Republic	28946,2	36813,2	40196,9	41076,4	46373,0	50416,9	55666,6	58917,6	60274,5
Arkhangelsk oblast	28474,7	33654,6	36224,0	38923,0	43223,3	48845,3	54129,0	59706,9	65862,1
Vologda oblast	26342,0	29364,2	31820,2	32944,5	36112,0	39485,1	45039,8	48422,7	51237,2
Kaliningrad oblast	20494,9	19389,5	22469,4	23344,7	25385,7	29557,2	36204,4	41987,3	45621,2
Leningrad oblast	26652,0	30607,8	33594,1	35231,9	43298,5	52749,7	59541,8	64886,4	68104,7
Murmansk oblast	29473,5	30449,0	31795,8	33518,7	35870,1	40146,2	43907,2	46975,2	51421,4
Novgorod oblast	14334,1	15113,3	16593,9	16656,3	17973,1	19848,0	22646,8	25117,1	29084,4
Pskov oblast	12839,1	14073,4	17198,5	19591,5	22516,6	24141,1	27632,7	27065,5	30544,0
St. Petersburg	137217,3	154599,3	173919,8	187756,0	221397,3	269592,0	302711,6	339657,6	375441,6

Source: authors' calculations according to the site of the Federal state statistics service and the collection of "Russia's regions 2009", used for converting the Consumer Price Index.

Wage fund, accrued to the workers on payroll and external by-workers (S), value of the index for the year in 2000 prices, million rubles

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	9023,7	9908,1	11810,4	12601,7	13811,5	15593,1	17479,9	19734,8	19576,9
Komi Republic	17941,0	21080,6	22999,0	24033,9	25476,0	27998,1	30853,9	33970,1	33970,1
Arkhangelsk oblast	15557,3	18139,8	20770,1	23013,3	25982,0	29047,9	31633,1	35239,3	34428,8
Vologda oblast	15712,4	18226,4	20377,1	22068,4	25202,1	28327,2	30905,0	34211,8	33698,6
Kaliningrad oblast	6276,2	7631,8	9539,8	10675,0	11198,1	12138,7	16108,1	19313,6	21380,1
Leningrad oblast	14458,3	17451,1	20958,8	22216,3	24082,5	26803,8	28948,1	34129,9	31263,0
Murmansk oblast	15980,6	18281,8	20274,5	20578,6	21566,4	23615,2	26401,8	29543,6	29514,1
Novgorod oblast	5196,2	6116,0	7351,4	8218,8	9131,1	10190,3	11841,2	13427,9	13750,2
Pskov oblast	4444,5	5253,4	6440,7	7393,9	7992,8	9031,9	9944,1	11813,6	10951,2
St. Petersburg	59776,9	71672,5	91167,4	95543,5	105862,1	118353,9	138829,1	168677,4	162942,3

Source: Authors' calculations according to the site of the Federal state statistics service and the collection of "Russia's regions 2009", we used Rosstat data on real wages as a percentage of the previous year for converting.

Average number of people employed in the economy (L), thousands of people

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	343,1	338,6	342,9	353,1	351,7	349	352,5	353,7	350,4
Komi Republic	459,9	466,4	463,9	468,9	473,5	474,9	475,4	476,1	475,4
Arkhangelsk oblast	594,4	601,3	609	609,7	600,4	601,5	618,9	621,4	615,1
Vologda oblast	622,6	623	623,9	619,2	600,4	605,7	610,4	613,7	611,3
Kaliningrad oblast	413,2	409,1	414	422,7	440,1	446,8	464,8	475,9	475,8
Leningrad oblast	710,6	715,8	718,3	725,3	733,6	736,6	743,4	748,7	743,6
Murmansk oblast	432,9	433,5	436,8	440,6	446	445,8	445,8	444	442,9
Novgorod oblast	318,1	316,3	315	313,8	314,9	313,9	310,5	313,6	316,3
Pskov oblast	331,1	327,5	323,8	326,5	338,7	336,6	334,3	333,9	332,6
St. Petersburg	2383,7	2397,2	2412	2410,2	2414,5	2427	2445,2	2473,4	2472,1
Source: Russian Regions 2009. – P. 106.									

Number of unemployed (B), thousands of people

Region	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
Republic of Karelia	43	33	30	32	28	33	14	24	32
Komi Republic	66	76	49	65	66	63	68	55	42
Arkhangelsk oblast	90	61	58	70	50	38	41	39	48
Vologda oblast	55	59	40	30	40	34	35	27	39
Kaliningrad oblast	77	39	35	38	33	34	23	18	47
Leningrad oblast	82	62	59	76	61	66	53	37	55
Murmansk oblast	73	69	55	53	56	46	35	33	36
Novgorod oblast	28	23	22	18	21	20	19	18	17
Pskov oblast	43	37	28	29	21	25	27	18	25
St. Petersburg	153	98	86	105	69	55	63	56	55
Source: Russian Regions 2009. – P. 130.									